

GINI COEFFICIENTS, SOCIAL NETWORK ANALYSIS, AND
MARKOV CHAINS: QUANTITATIVE METHODS FOR ANA-
LYZING THE DISTRIBUTION OF BENEFITS IN NATURAL
RESOURCE-DEPENDENT COMMUNITIES

by

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Abstract

The equitable distribution of benefits is a critical component of sustainable community-based management of natural resources. This paper measures the circulation of currency among businesses in a small, natural resource-dependence community in interior British Columbia, and shows how this could be tied to a quantified equitable distribution of extraction opportunities. As part of the development of the tools, this paper shows the equivalence of input-output models, social network analysis, and Markov chains. While ideologically the approach of quantifying equality is of interest to many, in reality and practice there are significant challenges to the adoption and implementation of these methods.

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1. Framing the Analysis

1.1 Community-based natural resource management

Human survival depends on the extraction of natural resources from the planet. These natural resources may be renewable organic material, such as trees, or non-renewable inorganic material, such as iron ore. Extraction is by necessity mildly disturbing to severely destructive of the environment surrounding the natural resource. While the extraction and processing of natural resources yields economic and social benefits to humans, this must be balanced against the environmental and social costs. There are pertinent questions about to whom do the benefits accrue and to whom (and to what) are costs incurred.

Management of natural resources has evolved from highly localized agrarian and maritime societies to broader kingdoms to transnational corporations ostensibly answerable to the governments of the countries within which they operate. The decision-making process for such global entities has a vastly different criteria for measuring benefits and costs than do the people residing next to the natural resource being exploited. In some locations local residents have retained control over those natural resources, often in the form of "the commons," such as grazing lands, forestry, and fisheries.

The commons, areas in which multiple parties can access, are often managed through "collective action." The local residents form the governance structure that limits access to the natural resource, through the development of rules that establish who can access the resource and how much they can extract. Long-enduring collective action systems can have decades of successful management, such as the case of the water rights in Raymond Basin in California, and even centuries, as with the irrigation systems of the Huertas in Spain (first documented in 1435, possibly 500 years older) and the Zanjeras in the Philippines (chartered in 1630) (Ostrom 1990). Conversely, previously successful collective action management systems with little or no environmental degradation have experienced almost immediate increased environmental degradation after nationalization or privatization. The nationalization of forests in Nepal led to increased deforestation for agriculture and exhibited substantial opportunistic behavior before the program was reversed (Bromley & Chapagain 1984). The privatization of land in Argentine Chaco led to absentee landownership that maximized profits at the expense of sustainable extraction rates (Altrichter & Basurto 2008). After millions of dollars were spent implementing a program to centralize irrigation decision-making in India, the yields per hectare became erratic and even declined (Ostrom, 1992).

A substantial amount of effort has been devoted to understanding the causes of successful and unsuccessful collective management of natural resources. Elinor Ostrom has focused on the design and "rules in use" aspects of the institution managing the natural resource, and has identified several components that are consistently present in successful collective action institutions (see Ostrom 1990, 1992, 2005). Evelyn Pinkerton has focused on "co-management" arrange-

ments between local communities and larger government bodies, and identified several rights that were devolved to the local management body in successful cases (see Pinkerton 1999; Pinkerton & Silver 2011; Pinkerton & Weinstein 1995). Both these two leaders in the field of community-based management of natural resources have collaborated with and inspired additional researchers, as the community-based management field now contains literally thousands of articles.

Within the concept of social equity in community-based management of natural resources are norms about the distribution of economic benefits within that community. An imbalance in the distribution of benefits leads to a relatively few achieving greater rewards than the balance of the community members, which can lead to a loss of social capital and trust, and can be a significant component to the unsustainable management of the resource (Andersson & Agrawal 2011). Equity issues appear in many cases studies on community-based management. Equity and equality are related but not the same, in that a certain degree of equality (sharing) is necessary to achieve equity (fairness). The issue of equity is of such importance that Ostrom (1992:69; 2010) identified equivalence between reward and effort as a key characteristics present in long-term sustainable community-based management. Pinkerton and Weinstein (1995:14) identify the right to allocate "access to a redistributive method" as necessary for successful community-based management of fisheries, and discuss how the case-study community would practice equitable "resource access or distribution." This paper endeavors to develop a method to quantify the equitable distribution of benefits in community-based management of natural resources, using a community forest in British Columbia as an example.

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1.2 Community-based forestry in British Columbia

In Canada, ownership of forested land is held by the provincial ("Crown"), and approximately 90% of the timber harvest Canada-wide is from public land (Sedjo 2005). In forest-dependent B.C. particularly, over 95% of the land is "Crown land" (Niquidet 2008). Although forest management in British Columbia has historically been the domain of the provincial government, through the use of volume-based tenures (B.C. MoFR 2006), community-based forest management has emerged in the late 1990s and early 2000s as an alternative approach. Pinkerton et al. (2008) and McCarthy (2006) each offer in-depth discussions of the political, economic and social factors that led to the establishment of a community-based forestry program in British Columbia.

Ambus, Davis-Case and Tyler (2007) document many governance aspects of community forestry in British Columbia, including managing for multiple values, and explain the alternate timber

pricing offered to community forests. Although there are many resemblances to the traditional forestry model (McCarthy 2006), a key difference is that community forests in B.C. are "area-based," instead of "volume-based" (Ambus & Hoberg 2011). Community forests are given a specific land area over which they have management rights, as opposed to the traditional model of allocation of a certain volume that can be harvested from Crown lands. Community-based management theory suggests that the community should determine the appropriate harvesting level (Ostrom 1990; Pinkerton and Weinstein 1995), but in B.C. this is set by the provincial government through the Timber Supply Review (B.C. MoFR 2005).

The earliest models of community forests in British Columbia (Mission and Revelstoke) used conventional Tree Farm Licenses (TFLs) and volume-based tenure agreements (Pinkerton et al. 2008; Gunter 2000). In 1998, the province of British Columbia initiated the Community Forests Pilot Program (McCarthy 2006). After an initial trial period, this program was formally implemented in 2005 and currently 44 community forest agreements have been finalized between the government of B.C. and community-based organizations, with an additional 12 nearing approval (BCCFA 2012). The Forest Revitalization Act of 2003 removed 20% of the total cut allocated to volume-based transnational corporations ("majors"), with defined breakdowns of how it would be reallocated among market-based sales, community forests, First Nations, and woodlot owners (Clogg 2003).

One critical change to B.C. forestry contained within the Forest Revitalization Act of 2003 was the elimination of the requirement that logs be processed within a certain distance from where they were harvested (Parker 2003), known in British Columbia as "appurtenancy" (Clogg 2003). As part of an exchange for access to public timber and reduced license costs, the forest corporations were proposed to have engaged in a "social contract" with the province of British Columbia to provide employment and environmental protections (Clogg 2003). This created many small towns centered around the mill, but was also identified as an impediment to economic efficiency at scales relevant to the transnational corporations that owned the mills (Crowe 2008). The elimination of appurtenancy has been identified as a serendipitous adaptation to the mountain pine beetle epidemic in British Columbia (Patriquin, Wellstead & White 2007), reinforcing Crowe's (2008) argument that sending "the right log to the right mill" will benefit local employment (in towns with mills that are still open). Surprisingly little academic research has focused on, or even considered, the impacts of the elimination of appurtenancy.

In addition to the desire to capture more economic benefits locally, community forests organizations were often formed in response to the logging practices near their communities by the majors. As such, a significant component of their *raison d'être* was protection of environmental values, including protecting drinking watersheds and critical wildlife habitat (Vernon 2007). Substantial control over the tenure area was theoretically "devolved" to the community forests (Ambus & Hoberg 2011), in addition to added responsibilities such as road construction (Gunter 2004).

1.3 The questions: how much does currency circulate in small communities, and how can benefits from local management of natural resources be distributed equitably?

A question facing community forests is whether to pursue the greatest sale price for their logs or provide greater access to logs locally, for value added enterprises such as local sawmills, log home builders, and other wood-related industries (Cathro 2004). Focusing on obtaining the greatest sale value for raw logs allows the community forest to use this money towards community-based projects (Mulkey & Gunter 2004). Alternatively, in recognizing that access to the natural resource is a benefit beyond the distribution of grants (Pinkerton et al. 2008), community forests could enhance the "multiplier effect" in creating direct and indirect economic benefits such as income and employment for local businesses. The harvesting of logs requires the purchase of fuel, which in turn requires an employee operating a store that sells the gas. These indirect contributions "multiply" the impact of the dollar spent within a community. If that gas is used to transport the raw logs to a distant community for milling, the opportunities for value-adding, such as local milling, are lost. That value-adding also has a multiplier effect, in terms of employment, products and services that are needed for the value-adding process.

Conceivably, grants vs. value-added may not be mutually exclusive, given certain compromises. Economies have "linkage" and "leakage" (Lejárraga & Walkenhorst 2010), where linkage is the connections between businesses exchanging currency and leakage is where currency slips out of the community through the purchase of products and services that cannot be provided locally. Grants and value-added opportunities create opportunities for both linkage and leakage. Understanding the magnitude of both can help communities determine which path to take, or what compromises to make.

That magnitude can be determined through mapping how a dollar circulates within a community. Under the right circumstances, this mapping could identify linkages and leakages within the community, allowing communities to adapt. Through reducing leakage and enhancing the multiplier effect, additional economic activity can occur. Alternatively, through the multiplier effect, the same level of economic activity within the community could occur while requiring less raw materials to do so. One focus of this paper is developing that mapping and applying it to a community in B.C. with a community forest tenure; the other focus is exploring quantitative methods for analyzing the distribution of benefits within community-based management of natural resources. Together they frame a quantitative analysis of equity.

1.4 Developing the tools

In this paper, three tools (social network analysis, Markov chains, and input-output models) are examined for their ability to calculate the average number of times a dollar circulates within a community before exiting. None of the three tools can do this calculation single-handedly, but together these tools offer a method for so doing.

Social network analysis (SNA) is a useful tool for analyzing community relationships (Knoke & Yang 2008). SNA data is built from identifying relationships between any two entities (nodes) within a system. In this case the relationship is the exchange of currency between two businesses. Markov chains are based on the probability of transitions between multiple states (Grinstead & Snell 1997:405). These transitions can be sequential, leading to the concept of "chains." This is comparable to a supply chain created from the production of a finished product from raw materials, where the "state" of the dollar is the business currently in possession of it. As businesses purchase products and services from other businesses, the dollar changes "states."

The input-output model was developed as a method of calculating the required output necessary by upstream industries to meet input needs of downstream industries as those downstream industries output changes (Miller & Blair 2009:1). The construction of a house creates a demand for products (outputs) such as lumber from upstream producers. This in turn creates an increase in demand for timber. At each location of demand, a "multiplier" occurs through indirect and induced impacts. Additional demand for logging increases, indirectly, demand for fuel and chainsaws. The total economic impact is not limited to the amount generated from the direct demand.

These three methods can be shown to be mathematically identical, allowing tools from each method to be used interchangeably with each other. In this paper, a small economy will be modeled as a game of Snakes and Ladders, with the connections crafted through social network analysis, and the average number of turns to complete the game (also known as the length of the game) is calculated through Markov Chains (Altheon, King & Schilling 1993). The length of the game, which in fact is the number of transactions a dollar takes from entering an economy until it exits, is then shown to be identical to the multiplier effect in input-output models, at a much more granular scale than the aggregation within industries used by input-output models.

As a means of quantifying the level of equality, as a component of equity, this paper will explore the Lorenz Curve and the Gini coefficient, two analytical methods for evaluating the distribution of an attribute within a population. Both of these tools are useful for quantifying different aspects of equality and equity. Neither has priority over the other, as they measure orthogonal attributes.

This paper then couples the average number of transactions a dollar takes to exit an economy from each individual business within the community and a limited unequal distribution using the Gini coefficient to create a method for distributing logging opportunities within a hypothetical community with a community forest. Although within this particular example the supply chain lengths are unrealistically long for the purpose of illustration, this paper will also discuss the results of surveying the economy of an actual community in rural British Columbia with a community forest tenure and provide the transaction chain lengths and the calculated average number of transactions a dollar takes within that community. A map of the community busi-

ness economy is also provided. Due to that community forest not yet logging, it was not possible to calculate the Gini coefficient of their logging opportunities.

1.5 Publications

Two publication streams are realizable with this paper. The first is a stream reminding economists and sociologists that input-output models and social network analysis tools are identical and that there should be greater cross-utilization between the fields. This is likely to generate two articles, one for economists and one for sociologists. The other stream is publishing the results of the economic mapping survey, as a means to answer the procedural question about how to measure the circulation of currency in a community.

1.6 Observations and conclusions

The surveying of the rural B.C. community revealed several practical obstacles to adopting the methods developed in this paper. The lack of industrial capacity within the community limited the ability to craft long transaction chains, and by extension, capture the maximum possible value for the raw material. The lack of professional capacity limited the ability for some businesses to participate in the survey.

If those obstacles can be overcome, community-based management of natural resources could potentially use the tools developed in this paper to quantify the equality of the distribution of benefits within their community. Suggestions on how to create incentives for local partnerships are included in the discussion. Through the use of these tools, community-based management groups can quantitatively justify rejecting higher prices for the extraction rights to the natural resource in favor of greater local access at lower prices and the enhanced collective outcome.

Each chapter within this paper contains greater detail on relevant literature and any necessary mathematical foundations for each step.

2. Circulation of currency

2.1 Examining the tools

Three tools (social network analysis, Markov chains, and input-output models) are examined for their ability to calculate the average number of times a dollar circulates within a community before exiting. None of the three tools can do this calculation single-handedly, but together these tools offer a method for so doing. Research into academic literature has failed to find other examples of using this approach, although some authors appear to come close. However, they do not complete the picture. For example, Kichiji and Nishibe (2008) use social network analysis to examine the circulation of community currency, an alternative to bank-backed money, but focus on the distribution of the currency flow, rather than the number of transactions within the community. Other authors (Hoekstra, van Arkel, & Leurs 2007; Horváth & Frechtling 1999) attempt to determine currency circulation using input-output models, but do not individualize businesses. Roberts (2005) even focuses on rural economies, and decomposes the currency flow using structural path analysis, coming very close to tracking how a dollar circulates in a rural economy, but this is by sector in an input-output model and not by business. At the risk of overgeneralizing, economic theory literature using Markov chains appears to focus on national levels, and does not examine local economies with granularity.

This chapter will show how the three tools are equivalent and can be used interchangeably. By using aspects of all three tools, this chapter will establish a method to map the circulation of currency in a small economy, and calculate the number of transactions that occur from the time a dollar enters the economy until it leaves. This method is exact and the correct one; all other methods are estimations.

2.1.1 Social Network Analysis

Social network analysis (SNA) is a useful tool for analyzing community relationships (Knoke and Yang 2008). SNA data is built from identifying relationships between any two entities, known as "nodes," within a system, such as people in a class. Connections between nodes can be directed (one way) or undirected (both ways), and are also known as "edges." The people in a class are the nodes, and if two people have a friendship, there is an edge between the two nodes. The connection strength may be a 0 (no connection) or 1 (connection), representing a binary relationship, or may range in values across any arbitrary scale (Hanneman & Riddle 2005; Knoke & Yang 2008; Wasserman & Faust 1994).

Social network data may take one of two forms. The data may be formatted on a line-by-line basis, where each line represents a connection between two nodes. This form has three field values: the originating node, the destination node, and the strength of the connection. This format, known as the "DL" format, is the most common method for storing SNA for analysis by software (Hanneman & Riddle 2005).

An alternative form uses a matrix, known as the "adjacency matrix" (Hanneman & Riddle 2005:Chapter 5). It represents nodes as rows, and the connections to other nodes are listed in the columns of each row, with each column representing the nodes in the system. There is no particular emphasis placed on the matrix form in social network analysis, as much of the emphasis is on the visualization of the network and the use of metrics relating to positions within the network by individual nodes.

There are several algorithms (methods) available for arranging the network being analyzed. The chosen algorithm is responsible for the different layouts seen in visualizations of networks. The Fruchterman-Reingold algorithm uses a "spring" to represent relationships between nodes, and attempts to display the system in an arrangement with the lowest potential energy. Yifan-Hu/Yifan-Hu Proportional algorithms use a "gravity" approach and pull more connected nodes to the center while pushing less connected nodes to the periphery. The Force Atlas algorithm also uses a gravity approach. Layouts can also be randomly displayed. Other layout algorithms are possible, as is manual arrangement for small systems (Knocke & Yang 2008; Wasserman & Faust 1994).

Hanneman and Riddle (2005), Knocke and Yang (2008) and Wasserman and Faust (1994) all present several metrics that are useful in quantifying values within a social network. Betweenness Centrality is the value for calculating how many connections pass through that node. Closeness Centrality is the value for determining how close a node is to the adjacent and non-adjacent nodes. Centrality can be affected by large numbers of connections within the network without differentiation with respect to specific nodes. Eccentricity is the reverse of Centrality, and measures how far away a node is from the other nodes. In Degree measures the number of directional connections into a node. Out Degree measures the number of directional connection out of a node. Total Degree measures the total number of connections in and out to a node. Degree can be affected by the sheer number of connections. While Degree and Centrality are useful attributes, they are insufficient for tracking currency flow within a community.

The shortest path length between two nodes, called the "geodesic," is often calculated for each pair of nodes in a network. This can be useful for locating clusters and subnetworks (Hanneman & Riddle 2005; Knocke & Yang 2008; Wasserman & Faust 1994). Social network analysis does not have a metric for calculating the average path length between two nodes, given more than one possible path (R. Hanneman, personal communication, Aug. 5, 2011).

There are several software packages that can take SNA data and render graphs and calculate the above metrics. Gephi (<http://www.gephi.org>) is a Java-based application that runs on all computer platforms with a Java engine.

Social network analysis has been used in many applications. It is experiencing a high level of interest in analyzing community-based management, as a method to analyze power relations and

clustering (see Ramirez-Sanchez & Pinkerton 2009; Lauber, Decker & Knuth 2008, e.g.). A search of academic literature yields thousands of published articles discussing and applying SNA, including health and diseases, crime, and supply chain logistics.

2.1.2 Markov chains

Markov chains are based on the 1907 work of A. A. Markov, who studied probability of transitions between multiple states (Grinstead & Snell 1997:405). These transitions can be sequential, leading to the concept of "chains." For example, an object may go from State A to State B to State C, or it may go from State A to State D. The probability of finding the object in State A, B, C, or D at any given point in time is the focus of Markov chain mathematics.

The transitions are usually formatted as a matrix, with each row indicating the probability of transitioning to a different state. If a state cannot be left once arrived at (the probability of transitioning to another state is 0), the Markov chain is known as an "absorbing" Markov chain (Grinstead & Snell 1997:416). There is a "canonical form" of transition matrices, with the states that can transition at the top of the matrix, and the absorbing states at the bottom. It is not necessary to include any state that an object may exist in prior to entering the system.

In analyzing the canonical form, only the transitional states are included. The absorbing states are omitted. Grinstead and Snell (1997:418) define a "fundamental matrix" of the form

$$(2.1.1) \quad N = [I - Q]^{-1}$$

where I is the identity matrix and Q is the matrix formed by the transition states. The average number of transitions from any state to an absorbing state and the number of times other states will be entered before reaching an absorbing state can be calculated from the elements in the rows of N .

Markov chains have been applied to a wide range of subjects, including queueing theory, ecological food webs, genetics, games, and information theory. Searches in academic literature databases on these topics typically yield hundreds if not thousands of returned articles. The use of Markov chains in the context of this paper will be explained in more detail later in this chapter.

2.1.3 Input-output models

The input-output model was developed in the "late 1930s" by Wassily Leontief, as a method of calculating the required output necessary by upstream industries to meet input needs of downstream industries as those downstream industries' output changes (Miller & Blair 2009:1). These demand requirements can be written as linear equations, representing the total demand for a given industry, and these linear equations can be expressed in matrix form. The Leontief inverse represents the "total requirements" of the included industries, recognizing their interdependence (Miller & Blair 2009:21).

$$(2.1.2) \quad L = [I - A]^{-1}$$

The Leontief inverse is constructed in part through the matrix of "technical coefficients" (Miller & Blair 2009:16), which is the ratio of the value of an input to the total value of that industry:

$$(2.1.3) \quad a_{ij} = \frac{\text{value of inputs from sector } i \text{ bought by sector } j}{\text{total value of the output of sector } j}$$

Specifically, given matrix A of technical coefficients and the identity matrix I :

$$(2.1.4) \quad A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n-1} & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n-1} & a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{n-11} & a_{n-12} & \cdots & a_{n-1n-1} & a_{n-1n} \\ a_{n1} & a_{n2} & \cdots & a_{n-1n-1} & a_{nn} \end{bmatrix}$$

$$(2.1.5) \quad I = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$(2.1.6) \quad [I - A]^{-1} = \begin{bmatrix} 1 - a_{11} & -a_{12} & \cdots & -a_{1n-1} & -a_{1n} \\ -a_{21} & 1 - a_{22} & \cdots & -a_{2n-1} & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -a_{n-11} & -a_{n-12} & \cdots & 1 - a_{n-1n-1} & -a_{n-1n} \\ -a_{n1} & -a_{n2} & \cdots & -a_{n-1n-1} & 1 - a_{nn} \end{bmatrix}^{-1}$$

One form of a "multiplier effect" can be calculated from the Leontief inverse. This form of the multiplier effect is the direct, indirect and induced increases in economic output necessary to support a given increase in output by a specific industry. If labor is one of these industries, the increased number of jobs can be calculated. Some of these jobs will directly come from the industry, some jobs will be indirectly created from industries that produce products used as raw materials (inputs) for the industry that is increasing its output, and some jobs will be "induced" through increases in these supplier industries.

Some notable differences between Markov chains and Leontief inverses exist. Leontief inverses do not have a concept of an "absorbing state," nor are the matrices organized in a specific form. Additionally, the matrix of technical coefficients is constructed vertically instead of horizontally as with Markov chains.

Input-output modeling has been applied to ecology, waste management, tourism, and many other venues. Input-output models are widely used in economic forecasting (Carnot, Koen, & Tissot 2005), but it is not without critics. Fundamentally the individual businesses are lost in the aggregation and there is no direct measurement of the currency circulation within a local economy (Cohen 1999). Policy makers rely on these input/output models for determining tax

breaks and other incentives to attract businesses to the local area but Moretti (2010) finds "little systematic evidence on the effects of successfully attracting a new firm on other parts of the local economy" as a result of this approach.

2.1.4 IO models, Social Network Analysis and Markov Chains are equivalent

In their development of Markov Chains, Ching and Ng (2006:3) define the transition matrix P to be the matrix form of $\sum p_{ij}$ where p_{ij} is the probability of transitioning to state j from state i . In contrast, Breuer and Baum (2005:9) define the probability p_{ij} to be from state i to state j , the inverse of Ching and Ng, but consistent with Bose (2002:149). Breuer and Baum (2005:81) refer to the "system of traffic equations," while Bose (2002:153) refers to "flow balance equations." These equations are the same equation that forms the basis of an input-output model, as given by Miller and Blair (2009:19). Bose also explicitly connects these equations to the matrix form that closely resembles the Leontief inverse. A notable difference is that the transition matrix is transposed, which would make it identical to the input-output model-based Leontief inverse but different than the SNA version. However, these methods result in identical calculations, as long as there is recognition as to whether the matrix was constructed horizontally (SNA-style) or vertically (input-output model-style). The relevant data is extracted from either the first row or first column, respectively. As a result, the average path length of a social network from point A to point B is identical to the average path length of a Markov chain and both are the same as the multiplier effect in input-output models (see Appendix A for a short proof). Since the average path length and the Leontief inverse are constructed and calculated in identical ways, and the Leontief inverse is a widely-accepted method for calculating the multiplier effect, the conclusion is clear: the average path length for a dollar entering a community until it exits is the same as the multiplier effect, on a more granular scale.

The identification of a correspondence between graph theory and economic modeling is attributed to "Koopmans [(1951)] and Morgenstern [(1954)]" in Asger Olsen (1992). Similarly, De-Genne and Forsé (1999) demonstrate that social network analysis is based in graph theory. Lesne (2006:239) highlights the "deep and operational relation between graph theory and Markov chain theory, the former providing demonstrative and constructive tools to the latter." Graph theory is fundamentally concerned with networks of all kinds, and utilizes matrices for calculations (Wallis 2007). It is, in fact, graph theory that unifies all three: input-output modeling, social network analysis, and Markov chains. This allows the tools from one field to be used in analyzing data from the other fields.

2.2 Measuring circulation of currency in a community

The above tools can be combined to develop a method to calculate the average number of times a dollar circulates in the community. This is done through a combination of mapping the economy using social network analysis and the use of Markov chains as described below. This allows the construction of a map of paths that a dollar may take from the point it enters the

community to the point it exits. For the purpose of mapping, the points of entry and exit can be represented by single nodes. Businesses are represented as nodes. The "bond" between nodes is the exchange of currency between two businesses. In order to utilize Markov chains, the exchange is in terms of the percentages of expenses going from a business to other businesses in the community. For example, business X may have 25% of its expenses going to business Y, 15% to business Z, and the remaining 60% exiting the community. It is not necessary to know the actual dollar amounts each business receives as income, as this can be modeled. The percentage of expense represents the probability of a dollar going to that downstream business.

Following the matrix form for social network analysis, the percentages will be in rows, with entry into the community in row 1, with business X's expense distributions in row 2, business Y in row 3, et al. Each column represents a downstream business, with column 1 occupied by entry into the community from outside (as a single node). Column 2 is business X, column 3 is business Y, et al. The last row and column represent exit from the community. For example, Eqn. 2.2.1 shows an expense matrix. Twenty percent of the total dollar amounts entering the community go to Business X, 50% go to Business Y, and 30% go to Business Z. Business X spends 35% of its expenses with Business Y, 45% with Business Z, and 30% outside of the local economy. All (100%) of Business Y's expenses are with Business Z, who in turn has all of its expenses with Business X. As all expenses going to the node marked "exit" remain with that node, "exit" is an "absorbing state," as per Markov chain definitions.

$$(2.2.1) \quad A = \begin{matrix} & \begin{matrix} enter & Bus.X & Bus.Y & Bus.Z & exit \end{matrix} \\ \begin{matrix} enter \\ Bus.X \\ Bus.Y \\ Bus.Z \\ exit \end{matrix} & \begin{pmatrix} 0 & 0.2 & 0.5 & 0.3 & 0 \\ 0 & 0 & 0.35 & 0.45 & 0.3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

As discussed above, the solution to finding the average path length of such a Markov chain is the Leontief inverse, with the row and column corresponding to exiting the community omitted (modeling exiting the community as an absorbing state). Once the Leontief inverse has been obtained, the sum of the first row of that Leontief inverse is the average path length of a dollar from entrance into the community until it exits.

A standard input-output model is insufficient for this approach. As noted earlier, it does not directly calculate the circulation of currency between businesses within an economy. Additionally, in small, rural communities, there may be only one or two businesses in the same industry.

2.3 Applying the tools

This paper will now document how to calculate the average number of times a dollar circulates in a small economy, using three different mathematical approaches. These approaches yield an identical result, showing that the approach chosen should depend on the degree of complexity of

the economy being analyzed. A simple four node loop with a recirculation loop will be used for illustrative purposes.

2.3.1 A simple recirculation loop

The simplest loop worth examining consists of three nodes, plus one node representing the departure from the local economy. A two node loop is merely a net exchange and not worth examining. Figure 2.1 below depicts this simple loop. The blue circle is a business at the start of the loop and the red circle is where the dollar leaves the community. This is equivalent to 100% of all currency entering an economy going to a single business.

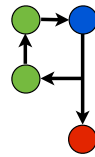


Figure 2.3.1 Simple Four-node loop

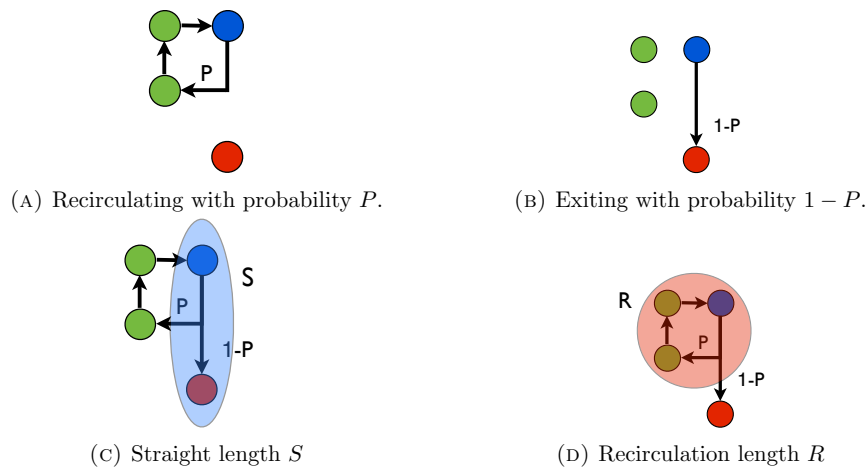


Figure 2.3.2 Components of a simple four node loop

Let P be the probability that the dollar will circulate through the loop (see Figure 2.3.2a), while $1-P$ is the probability the dollar will escape (see Figure 2.3.2b). Let S be the length from the blue circle to the red circle (see Figure 2.3.2c, S is equal to 1), and let R be the length around the loop of green circles (See Figure 2.3.2d, R is equal to 3). The path length L is the average number of times a dollar circulates before escaping to the red circle. This is calculated from the weighted probability the dollar escapes. A dollar that escapes contributes its average path length times the probability it escapes. Each dollar has a probability P of looping each time, times the path length of the loop. For example, if a dollar escapes directly (0 loops), its average

path length is

$$(2.3.1) \quad L_0 = (1 - P) \cdot S$$

If the dollar circulates once and then escapes, its average path length is the sum of the length of the loop (R) plus the length of the path to escape (S) (the total number of nodes it passed through before escaping), times the probability P it looped one time, times the probability $1-P$ it escaped after that one loop:

$$(2.3.2) \quad L_1 = (1 - P) \cdot P \cdot (R + S)$$

A dollar has a probability $P \cdot P$ of circulating for a second loop, but escaping with probability $1-P$. If it does escape after two loops, its average path length is

$$(2.3.3) \quad L_2 = (1 - P) \cdot P \cdot P \cdot (2 \cdot R + S)$$

While the recirculation loop R is included twice, for the two loops, the probability $P \cdot P$ of looping twice shows a decreasing contribution to the total average path length. This continues indefinitely, as there is a non-zero probability of continuing to loop, even after circulating a large number of times. The total path length (denoted \bar{L}) is the sum of average path loops, up to an infinite number of loops.

$$(2.3.4) \quad \bar{L} = L_0 + L_1 + L_2 + L_3 + L_4 + \cdots + L_\infty$$

$$(2.3.5) \quad \begin{aligned} \bar{L} &= (1 - P) \cdot S \\ &+ (1 - P) \cdot P \cdot (R + S) \\ &+ (1 - P) \cdot P \cdot P \cdot (2 \cdot R + S) \\ &+ (1 - P) \cdot P \cdot P \cdot P \cdot (3 \cdot R + S) \\ &+ \cdots \\ &+ (1 - P) \cdot P^\infty (\infty \cdot R + S) \end{aligned}$$

This equation can be reduced to a summation, recognizing that $(1-P)$ can be factored out, and that each loop is a power of P and a multiple of R :

$$(2.3.6) \quad \bar{L} = (1 - P) \cdot \sum_{n=0}^{\infty} [P^n \cdot (Rn + S)]$$

Solving this requires distributing the P^n term and determining the value for the two summations, pulling out the constants R and S :

$$(2.3.7) \quad \bar{L} = (1 - P) \cdot [R \cdot \sum_{n=0}^{\infty} nP^n + S \cdot \sum_{n=0}^{\infty} P^n]$$

These two geometric series converge for all $P < 1$. Let r equal the series on the left and s equal the series on the right:

$$(2.3.8) \quad r = \sum_{n=0}^{\infty} nP^n$$

$$(2.3.9) \quad s = \sum_{n=0}^{\infty} P^n$$

Solving s first is easier and the results will be used to solve r .

$$(2.3.10) \quad s = \sum_{n=0}^{\infty} P^n = 1 + P + P^2 + P^3 + P^4 + \dots + P^{\infty}$$

$$(2.3.11) \quad P \cdot s = P \cdot \sum_{n=0}^{\infty} P^n = \sum_{n=0}^{\infty} P^{n+1} = P + P^2 + P^3 + P^4 + \dots + P^{\infty}$$

$$s - P \cdot s = 1 + [P - P] + [P^2 - P^2] + [P^3 - P^3] \\ + [P^4 - P^4] + \dots + [P^{\infty} - P^{\infty}]$$

$$(1 - P) \cdot s = 1$$

$$s = \frac{1}{1 - P}$$

$$(2.3.12) \quad \sum_{n=0}^{\infty} P^n = \frac{1}{1 - P}$$

Continuing with r :

$$(2.3.13) \quad r = \sum_{n=0}^{\infty} n \cdot P^n = P + 2P^2 + 3P^3 + 4P^4 + \dots + \infty \cdot P^{\infty}$$

$$(2.3.14)$$

$$P \cdot r = P \cdot \sum_{n=0}^{\infty} n \cdot P^n = \sum_{n=0}^{\infty} n \cdot P^{n+1} = P^2 + 2P^3 + 3P^4 + \dots + (\infty - 1)P^{\infty}$$

$$r - P \cdot r = P + [2P^2 - P^2] + [3P^3 - 2P^3] \\ + [4P^4 - 3P^4] + \dots + [\infty \cdot P^{\infty} - (\infty - 1) \cdot P^{\infty}] \\ = P + P^2 + P^3 + P^4 + \dots + P^{\infty}$$

$$= \sum_{n=0}^{\infty} P^n - 1$$

$$= \frac{1}{(1 - P)} - 1 \quad \text{Using (2.3.12)}$$

$$= \frac{1 - (1 - P)}{1 - P}$$

$$= \frac{P}{1 - P}$$

$$r \cdot (1 - P) = \frac{P}{1 - P}$$

$$r = \frac{P}{(1 - P)^2}$$

$$(2.3.15) \quad \sum_{n=0}^{\infty} n \cdot P^n = \frac{P}{(1 - P)^2}$$

Using (2.3.12) and (2.3.15), (2.3.7) can now be solved exactly.

$$\begin{aligned}
(2.3.7) \quad \bar{L} &= (1 - P) \cdot \left[R \cdot \sum_{n=0}^{\infty} nP^n + S \cdot \sum_{n=0}^{\infty} P^n \right] \\
&= (1 - P) \cdot \left[R \cdot \frac{P}{(1 - P)^2} + S \cdot \frac{1}{1 - P} \right] \\
&= (1 - P) \cdot R \cdot \frac{P}{(1 - P)^2} + (1 - P) \cdot S \cdot \frac{1}{1 - P} \\
(2.3.16) \quad \bar{L} &= \frac{P}{1 - P} \cdot R + S
\end{aligned}$$

The average path length (the number of times a dollar circulates) in the above four-node loop is the length of the loop times the ratio of the probability the dollar will recirculate to the probability it escapes, plus the length of the escape.

2.3.2 Social network analysis does not handle loops

Social network analysis has one measurement related to path length, that of the geodesic path (Knoke & Yang 2008). This path length is the shortest distance between two nodes. In the case of the four-node loop, the shortest distance between the blue circle (a business at the start of the loop) and the red circle (leaving the community) in Figure 1 is a straight path, of distance 1. The contribution by the loop is not considered. A review of the books on social network analysis by Wasserman and Faust (1994), Knoke and Yang (2008), and Friemel (2008) lack any mention of calculating the impact of loops, and only Friemel actually mentions loops at all. As discussed above, tools from the other two fields, input-output models and Markov chains, can be used and both of them regularly deal with loops.

2.3.3 A local economy as a Markov chain

The four-node loop can be represented as a matrix, identical in form to the matrix form found in social network analysis (see Section 2.2). Instead of a binary 0 or 1, the strength of the connection between the two nodes is a number between 0 and 1, representing the probability of a dollar going to the downstream node. Exiting the loop is represented as an absorbing state, with a probability of 1 that the state will transition to itself. Each row in the matrix is a business, with the exception of the last one, which is exiting the loop. Each column is also a business, downstream of the business represented in the row. For example, Business X has a probability P of spending a dollar locally with Business Y, and a probability $1 - P$ of spending a dollar with a business outside of the local area. See Figures 2.1 and 2.2 for a graphical illustration of such a loop, with Business X represented by the blue circle, Business Y and Z represented by the two green circles, and leaving the community represented by the red circle.

$$(2.3.17) \quad A = \begin{array}{c} \text{Bus.X} \\ \text{Bus.Y} \\ \text{Bus.Z} \\ \text{exit} \end{array} \begin{array}{cccc} & \text{Bus.X} & \text{Bus.Y} & \text{Bus.Z} & \text{exit} \\ \left(\begin{array}{cccc} 0 & P & 0 & 1 - P \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) \end{array}$$

This can be extended indefinitely. Conceptually, a localized economy resembles a game of Snakes and Ladders, where players roll a dice and have a probability of landing on a ladder and advancing several rows, landing on a snake and moving backwards several rows, or missing both and advancing linearly along the path. The average number of turns it takes to complete a game of Snakes and Ladders is the same as the average number of transactions it takes for a dollar to exit a community. The game Snakes and Ladders has been "solved" by constructing a Markov chain matrix and calculating the fundamental matrix (Altheon, King and Schilling 1993), which is equivalent to the Leontief inverse.

2.3.4 Calculating the Leontief inverse

A common method for finding the Leontief inverse is to use the determinant and adjunct of the matrix constructed from $(I-A)$ (Miller & Blair 2009:23,693). As discussed in Appendix B, the formula for calculating an inverse of a matrix B is

$$(B.4) \quad B^{-1} = \frac{1}{\det|B|} \text{adj}(B)$$

For the Leontief inverse, this means

$$(2.3.18) \quad L = [I - A]^{-1} = \frac{1}{\det|[I - A]|} \text{adj}([I - A])$$

For large matrices this can be quite computationally intensive. Alternative methods to finding the Leontief inverse are discussed below.

2.3.5 McLaurin series expansion

An alternative method for obtaining the Leontief inverse is to use a McLaurin series expansion (Miller & Blair 2009:244). This is based on the solution to finding $1/x$ where x is less than one:

$$(2.3.19) \quad \frac{1}{x} = 1 + x + x^2 + x^3 + \dots$$

If the Leontief inverse is normalized, all values in L will be less than one, and therefore the inverse can be calculated through the expansion:

$$(2.3.20) \quad L = [I - A]^{-1} = I + A + A^2 + A^3 + \dots$$

where A^2 represents the matrix multiplication $A \times A$, and so on.

This form is the same form used to derive the "fundamental matrix" for an absorbing Markov

chain in Grinstead and Snell (1997:419), hinting of parallels between input-output models and Markov chains discussed earlier.

Example: Four-Node Loop with McLaurin Series expansion

An example calculating the average path length of a four-node loop using the Taylor Series expansion will now be provided. Three businesses will be used, Business X, Business Y, and Business Z. Business X has a percentage P going to Business Y, while $1-P$ leaves the community. Business Y has 100% of its expenses going to Business Z, and 100% of Business Z's expenses go to Business X. Refer to the diagrams in Figures 2.1 and 2.2 for a graphical image of the four-node loop. As discussed earlier, the Markov chain matrix form of the four-node loop is:

$$(2.3.21) \quad A = \begin{bmatrix} 0 & P & 0 & 1-P \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and the identity matrix I is

$$(2.3.22) \quad I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

However, as discussed earlier, the last row and column will be omitted, corresponding with the absorbing state of exiting the community. As such, the reduced matrices are

$$(2.3.23) \quad A = \begin{bmatrix} 0 & P & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$(2.3.24) \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For this example, the multiples of A through A^6 will be calculated (the individual multiplications will be omitted after A^2).

$$(2.3.25) \quad \begin{aligned} A^2 = A \cdot A &= \begin{bmatrix} 0 & P & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & P & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} (0 \cdot 0) + (P \cdot 0) + (0 \cdot 1) & (0 \cdot P) + (P \cdot 0) + (0 \cdot 0) & (0 \cdot 0) + (P \cdot 1) + (0 \cdot 0) \\ (0 \cdot 0) + (0 \cdot 0) + (1 \cdot 1) & (0 \cdot P) + (0 \cdot 0) + (1 \cdot 0) & (0 \cdot P) + (0 \cdot 0) + (1 \cdot 0) \\ (1 \cdot 0) + (0 \cdot 1) + (0 \cdot 0) & (1 \cdot P) + (0 \cdot 0) + (0 \cdot 0) & (1 \cdot 0) + (0 \cdot 1) + (0 \cdot 0) \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & P \\ 1 & 0 & 0 \\ 0 & P & 0 \end{bmatrix} \end{aligned}$$

$$(2.3.26) \quad A^3 = A^2 \cdot A = \begin{bmatrix} 0 & 0 & P \\ 1 & 0 & 0 \\ 0 & P & 0 \end{bmatrix} \begin{bmatrix} 0 & P & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{bmatrix}$$

$$(2.3.27) \quad A^4 = A^3 \cdot A = \begin{bmatrix} P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{bmatrix} \begin{bmatrix} 0 & P & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & P^2 & 0 \\ 0 & 0 & P^2 \\ P^2 & 0 & 0 \end{bmatrix}$$

$$(2.3.28) \quad A^5 = A^4 \cdot A = \begin{bmatrix} 0 & P^2 & 0 \\ 0 & 0 & P^2 \\ P^2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & P & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & P^2 \\ P & 0 & 0 \\ 0 & P^2 & 0 \end{bmatrix}$$

$$(2.3.29) \quad A^6 = A^5 \cdot A = \begin{bmatrix} 0 & 0 & P^2 \\ P & 0 & 0 \\ 0 & P^2 & 0 \end{bmatrix} \begin{bmatrix} 0 & P & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} P^2 & 0 & 0 \\ 0 & P^2 & 0 \\ 0 & 0 & P^2 \end{bmatrix}$$

Further calculations can be done, but a pattern is evident at this point. Every three cycles the probability P increases in power, from P to P^2 to P^3 and so on. Summing the first rows of each of the calculated matrices to obtain the average path length:

$$(2.3.30) \quad \begin{aligned} \bar{L} &= I_{1j} + A_{1j} + A_{1j}^2 + A_{1j}^3 + A_{1j}^4 + A_{1j}^5 + A_{1j}^6 + \dots + A_{1j}^\infty \\ &= 1 + P + P + P + P^2 + P^2 + P^2 + \dots + P^\infty + P^\infty + P^\infty \end{aligned}$$

The terms that cycle as a multiple of the number of nodes in the loop can be grouped:

$$(2.3.31) \quad \begin{aligned} \bar{L} &= 1 + 3[P + P^2 + \dots + P^\infty] \\ &= 1 + 3 \cdot \sum_{n=1}^{\infty} P^n \end{aligned}$$

The summation can be expanded to include $n=0$:

$$(2.3.32) \quad \bar{L} = 1 + 3 \cdot \left(\sum_{n=0}^{\infty} P^n - 1 \right)$$

Using (2.3.12) from the geometric series solution earlier in the chapter, the summation can be reduced, and further solved:

$$(2.3.33) \quad \begin{aligned} \bar{L} &= 1 + 3 \cdot \left(\frac{1}{1-P} - 1 \right) \\ &= 1 + 3 \cdot \left(\frac{1}{1-P} - \frac{1-P}{1-P} \right) \\ &= 1 + 3 \cdot \left(\frac{1-1+P}{1-P} \right) \\ &= 1 + 3 \cdot \left(\frac{P}{1-P} \right) \\ \bar{L} &= \left(\frac{P}{1-P} \right) \cdot 3 + 1 \end{aligned}$$

This is the same result as derived from the geometric series solution, with a loop size R of 3 and an exit length S of 1. With larger matrices, this method rapidly becomes quite unwieldy. For a matrix of dimension n , there are $n \times n$ calculations for each power of A . Additionally, when using numerical methods, this approach converges slowly. However, it has an advantage of being associated with time, in that powers of P above 1 are secondary indirect effects. This has been

used to describe flows within the economy, as structural path analysis (see Miller & Blair 2009:244; Defourny & Thorbecke 1984; Duchin & Levine 2010). A full discussion of structural path analysis is beyond the scope of this paper, but could be explored in future work as it appears there are overlaps.

2.3.5 Gauss-Jordan Elimination

A third method for obtaining the Leontief inverse is to use a technique known as Gauss-Jordan Elimination (McMahon 2006:27. Gauss-Jordan Elimination provides a means for using row and column manipulation to find the inverse of a matrix. Given a matrix formed by merging matrix A (Eqn. 2.1.4) with the identity matrix I (Eqn. 2.1.5),

$$(2.3.34) \quad [AI] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n-1} & a_{1n} & 1 & 0 & \cdots & 0 & 0 \\ a_{21} & a_{22} & \cdots & a_{2n-1} & a_{2n} & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{n-11} & a_{n-12} & \cdots & a_{n-1n-1} & a_{n-1n} & 0 & 0 & \cdots & 1 & 0 \\ a_{n1} & a_{n2} & \cdots & a_{n-1n-1} & a_{nn} & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

the reduction of the a elements until they form the identity matrix I yields the inverse of A :

$$(2.3.35) \quad [IA^{-1}] = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & a_{11}^{-1} & a_{12}^{-1} & \cdots & a_{1n-1}^{-1} & a_{1n}^{-1} \\ 0 & 1 & \cdots & 0 & 0 & a_{21}^{-1} & a_{22}^{-1} & \cdots & a_{2n-1}^{-1} & a_{2n}^{-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & a_{n-11}^{-1} & a_{n-12}^{-1} & \cdots & a_{n-1n-1}^{-1} & a_{n-1n}^{-1} \\ 0 & 0 & \cdots & 0 & 1 & a_{n1}^{-1} & a_{n2}^{-1} & \cdots & a_{n-1n-1}^{-1} & a_{nn}^{-1} \end{bmatrix}$$

This is due to the associative property of linear algebra:

$$(2.3.36) \quad \underbrace{[AI]}_a A^{-1} = A \underbrace{[IA^{-1}]}_{a^{-1}} = I$$

The Gauss-Jordan Elimination method is very useful in finding the inverses of matrices using computers (Sewell 2005:57).

Example: Four-Node Loop with Gauss-Jordan Elimination

As an example showing a proof of concept, the previously discussed four-node loop will be constructed as a Markov chain and solved with Gauss-Jordan elimination. As previously discussed in the example using the McLaurin series expansion, the matrix A is

$$(2.3.21) \quad A = \begin{bmatrix} 0 & P & 0 & 1-P \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The identity matrix for a 4x4 matrix:

$$(2.3.22) \quad I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Constructing the Leontief inverse:

$$(2.3.37) \quad [I - A] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & P & 0 & 1-P \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -P & 0 & P \\ 0 & 1 & -1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

As explained above, the last column and row are dropped, and the inverse is taken of the remaining matrix:

$$(2.3.38) \quad L = [I - A]^{-1} = \begin{bmatrix} 1 & -P & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}^{-1}$$

Gauss-Jordan Elimination uses the identity matrix to expand the matrix for which the inverse is being found (in this case $[I - A]$):

$$(2.3.39) \quad [I - A][I - A]^{-1} = [[I - A]I]L = \begin{bmatrix} 1 & -P & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} L$$

For clarity, only the matrix will be displayed, along with the matrix manipulation necessary at each step in order to achieve the identity matrix in the left hand side of the matrix.

Add row 1 to row 3:

$$(2.3.40) \quad \begin{bmatrix} 1 & -P & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 1-1 & 0-P & 1+0 & 0+1 & 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 1 & -P & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & -P & 1 & 1 & 0 & 1 \end{bmatrix}$$

Multiply row 2 by R and add it to row 3:

$$(2.3.41) \quad \begin{bmatrix} 1 & -P & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & -P+P & 1-P & 1+0 & 0+P & 1+0 \end{bmatrix} = \begin{bmatrix} 1 & -P & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1-P & 1 & P & 1 \end{bmatrix}$$

Multiply row 3 by $1/1-P$:

$$(2.3.42) \quad \begin{bmatrix} 1 & -P & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 \cdot \frac{1}{1-P} & 0 \cdot \frac{1}{1-P} & 1-P \cdot \frac{1}{1-P} & 1 \cdot \frac{1}{1-P} & P \cdot \frac{1}{1-P} & 1 \cdot \frac{1}{1-P} \end{bmatrix} = \begin{bmatrix} 1 & -P & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{1-P} & \frac{P}{1-P} & \frac{1}{1-P} \end{bmatrix}$$

Add row 3 to row 2:

$$(2.3.43) \quad \begin{bmatrix} 1 & -P & 0 & 1 & 0 & 0 \\ 0+0 & 1+0 & -1+1 & 0+\frac{1}{1-P} & 1+\frac{P}{1-P} & 0+\frac{1}{1-P} \\ 0 & 0 & 1 & \frac{1}{1-P} & \frac{P}{1-P} & \frac{1}{1-P} \end{bmatrix} = \begin{bmatrix} 1 & -P & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{1-P} & \frac{1}{1-P} & \frac{1}{1-P} \\ 0 & 0 & 1 & \frac{1}{1-P} & \frac{P}{1-P} & \frac{1}{1-P} \end{bmatrix}$$

Multiply row 2 by R and add it to row 1:

$$(2.3.44)$$

$$\begin{bmatrix} 1+0 & -P+P & 0+0 & 1+\frac{P}{1-P} & 0+\frac{P}{1-P} & 0+\frac{P}{1-P} \\ 0 & 1 & 0 & \frac{1}{1-P} & \frac{1}{1-P} & \frac{1}{1-P} \\ 0 & 0 & 1 & \frac{1}{1-P} & \frac{P}{1-P} & \frac{1}{1-P} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1+\frac{P}{1-P} & \frac{P}{1-P} & \frac{P}{1-P} \\ 0 & 1 & 0 & \frac{1}{1-P} & \frac{1}{1-P} & \frac{1}{1-P} \\ 0 & 0 & 1 & \frac{1}{1-P} & \frac{P}{1-P} & \frac{1}{1-P} \end{bmatrix}$$

This now leaves the left hand side equal to the identity matrix I, and therefore the right hand side is the inverse of matrix L :

$$(2.3.45) \quad L = \begin{bmatrix} 1+\frac{1}{1-P} & \frac{1}{1-P} & \frac{1}{1-P} \\ \frac{1}{1-P} & \frac{1}{1-P} & \frac{1}{1-P} \\ \frac{1}{1-P} & \frac{P}{1-P} & \frac{1}{1-P} \end{bmatrix}$$

Summing the first row:

$$(2.3.46) \quad \sum_{j=1}^3 l_{1j} = 1 + \frac{P}{1-P} + \frac{P}{1-P} + \frac{P}{1-P} = \frac{P}{1-P} \cdot 3 + 1$$

Using the Gauss-Jordan Elimination method, the average path length \bar{L} is identical to that of the geometric solution in Appendix A, with a loop size R of 3 and an exit length S of 1:

$$(2.3.47) \quad \bar{L} = \frac{P}{1-P} \cdot 3 + 1$$

This shows that the Gauss-Jordan method can be used to calculate the average path length of an economy. It has fewer calculations than either the geometric series or McLaurin series expansion.

2.4 The impact of loops

While local value-added efforts will increase the average number of transactions for a dollar entering until it leaves a community, a potentially greater impact is that of recirculating dollars through loops in the economy.

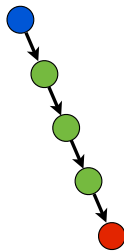


Figure 2.4.1a: Five nodes in a chain

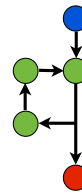


Figure 2.4.1b: Five nodes with a loop

Consider two economic chains, with an equal number of businesses (see Figure 2.4.1a and 2.4.1b). The blue circle represents currency entering the community and the red circle represents it leaving. For the chain represented in Figure 2.4.1a, assuming each business spends 100% of its expenses with the next business in the chain, the total economic activity is the sum of the transactions. If each transaction is \$100, and each arrow represents a transaction, the total economic activity is \$400 (dollars flow in the direction of the arrows, products and services

flow in the opposite direction).

For the chain represented in Figure 2.4.1b, the situation is different. As discussed earlier in this chapter, a percentage of dollars recirculate through the loop, and the balance escapes. If the percentage of recirculation is 50%, half of the dollars escape during each period of time, but half recirculate. Although the economic impact of this recirculation diminishes as half escapes each loop, the contribution to the economic activity remains for many loops.

Using the formula calculated above (Eqn. 2.3.16), the impact of recirculating dollars can be quantified.

$$(2.3.16) \quad \bar{L} = \frac{P}{1-P} \cdot R + S$$

There are three nodes in the loop R , and the length S is two (from blue to green to red). For a probability of recirculation P of .5, the average path length (multiplier) is 5:

$$(2.4.1) \quad \bar{L} = \frac{.5}{1-.5} \cdot 3 + 2 = \frac{.5}{.5} \cdot 3 + 2 = 5$$

Using \$100 for the initial transaction (entering the community), the total economic activity (over time) is \$500. Alternatively, if the initial transaction is \$80, the total economic activity is \$400 - the same as the total economic activity in Figure 2.4.1a, but with a lower input. The ability to maintain the same level of economic activity in spite of a lower input means greater resilience of decreased global economic activity. Additionally, it also means the same amount of economic activity while requiring less raw materials.

Although the total economic activity of a loop requires a longer time period to achieve, this economic activity persists in the absence of continuing inputs from outside the community. Communities dependent on natural resource extraction should undertake building loops within the community as a mitigation effort against the frequent boom and bust cycles prevalent in global and regional natural resource commodity markets (Clapp 1998).

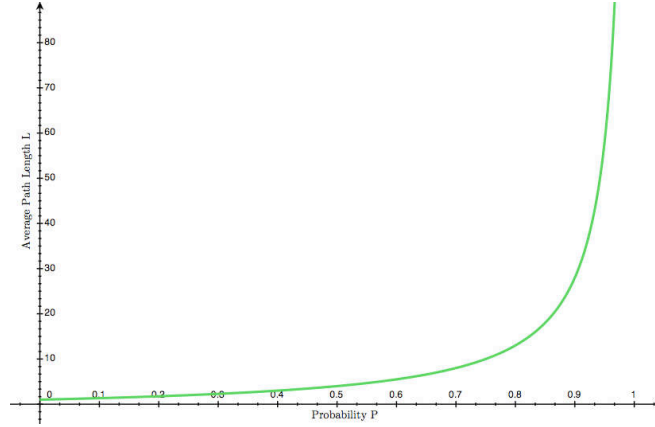


Figure 2.4.2: Average path length L as a function of P

Figure 2.4.2 graphically demonstrates the non-linearly increasing average path length L as the probability of recirculation in the four node loop increases. At $P = 50\%$, the average path length is 5, while at $P = 75\%$, the average path length is 10.

Conversely, Eqn. 2.3.16 can be solved to find the needed inputs to maintain the same output as a function of recirculation, using the multiplier effect in loops to enhance the reduced inputs. Figure 2.4.1b, five nodes with a loop, has a total number of transaction opportunities $R+S$, in this $R=3$, and $S=2$ (refer to Figure 2.3.1c and 2.3.1d for identifying R and S). If there is no recirculation, there are a total of two transactions. Figure 2.4.1a has one less transaction opportunity, or $R+S-1$. Specifically, let T' be the economic activity of Figure 2.4.1a (straight chain), calculated by multiplying the path length times i' , the input amount:

$$(2.4.2) \quad T' = L' \cdot i' = (R + S - 1) \cdot i'$$

and let T be the economic activity of Figure 2.4.1b (loop), calculated by the average path length of the loop times i , the input amount:

$$(2.4.3) \quad T = L \cdot i = \left(\frac{P}{1-P} \cdot R + S\right) \cdot i$$

Finding the needed inputs to create the same output (or number of transactions) as a function of probability requires comparing the ratio of economic activity for the two arrangements, and setting the ratio equal to one (equal economic activity):

$$(2.4.4) \quad \frac{T'}{T} = \frac{(R + S - 1) \cdot i'}{\left(\frac{P}{1-P} \cdot R + S\right) \cdot i} = 1$$

If i' is assumed to be one dollar (1), i is the fraction of a dollar needed to maintain the ratio of economic activity for different P :

$$(2.4.5) \quad i = \frac{(R + S - 1)}{\left(\frac{P}{1-P} \cdot R + S\right)}$$

This can be reduced through algebraic steps:

$$(2.4.6) \quad i = \frac{(1 - P)(R + S - 1)}{P(R - S) + S}$$

Using $R=3$ and $S=2$, this is further reduced to:

$$(2.4.7) \quad i = \frac{(1-P)(3+2-1)}{P(3-2)+2} = \frac{(1-P) \cdot 4}{P+2}$$

Figure 2.4.3 graphs the needed inputs i as a function of the probability P of recirculating a dollar within the loop. With 0% recirculation, the loop in Figure 2.4.1b requires twice the inputs to match the economic activity of the straight chain in Figure 2.4.1a. This is consistent with only two transaction opportunities instead of four. At 40% recirculation, the inputs needed are equivalent; at 50% recirculation only 80% of the original input level is needed, and at 80% recirculation only 1/3 of the original inputs are needed to match the economic activity of the straight chain. An economy that is recirculating 80% of its dollars is a rather self-contained economy.

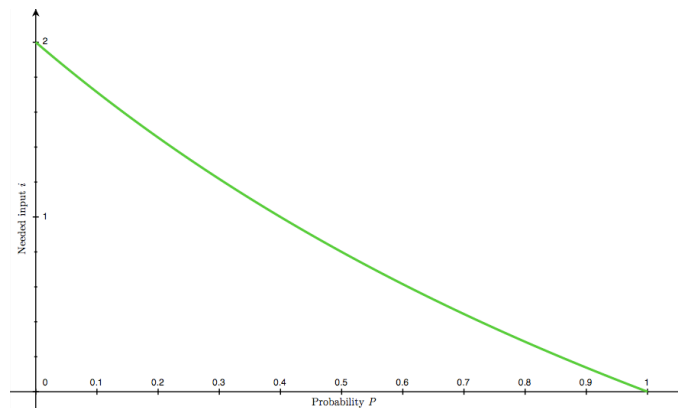


Figure 2.4.3: Needed inputs for equivalent outputs as a function of P

This can be extended to longer business supply chains, allowing an association between pricing for a resource and the multiplier effect for a given business. In the example above, reducing the price of the resource by 20% yields the same economic impact for a chain with an average path length of 5 as it does at 100% for a chain with an average path length of 4. This can be used to create incentives for local businesses to engage in greater utilization of other local businesses in their supply chain.

2.5 Chapter Summary

Through the use of equivalent graph theory-based tools found in social network analysis, Markov chains and input-output models, it is possible to map an economy and calculate on average how often a dollar is exchanged in a community before it leaves that community. Social network analysis is used to create a visual of the community's economy, while the "fundamental matrix" of Markov chains, also known as the "Leontief inverse" in input-output models, is used to calculate the average length of the path from the node that represents entrance into the community to the node that represents the exit from the community. The data necessary for this can be collected through surveys.

Chapter 4 discusses a small, rural natural resource dependent community in Interior British Co-

lumbia where this survey and method was carried out. Prior to that, though, Chapter 3 will discuss a method for quantitatively analyzing the distribution of benefits from managing a natural resource.

3. Distribution of resources/Gini coefficient

3.1 Equity and equality in community-based management

While substantial literature has been devoted to the successes of community-based natural resource management, some critical analysis of community-based natural resource management shows failures do occur, in sufficient numbers that some authors have expressed concern about the quality of research by those espousing successes (see e.g., Bradshaw 2003, Castree 2011). One area worthy of particular scrutiny is the distribution of benefits of community-based management. Chapter 1 introduced the concept of social equity in community-based management. Within the concept of social equity are norms about the distribution of economic benefits within the community. An imbalance in the distribution of benefits leads to relatively few community members achieving greater rewards than the balance of the collective, which can lead to a loss of social capital and trust, and can be a significant component in the unsustainable management of the resource (Andersson & Agrawal 2011).

The issue of equity is of such importance that Nobel Prize-winning scholar Dr. Elinor Ostrom included it as a design principle, making equivalence between reward and effort one of the key characteristics present in long-term sustainable community-based management (Ostrom 1992:69; 2010). Pinkerton and Weinstein (1995) identify the right to allocate internally, using community norms or rules, as necessary for successful community-based management of fisheries, and discuss how the case-study community would practice equitable "resource access or distribution." McDermott (2009:250) builds a framework around equity in analyzing community-based forestry (CBF), positing that "CBF initiatives will bring about social change when they transform the distribution of access to resources and decision-making power and scope." (McDermott 2009:250) further observes that "In order to reduce inequity, community-based organizations must make social equity an explicit target to which they hold themselves accountable."

Equity issues appear in many case studies on community-based management. Pinkerton and Edwards (2009) identify the inequitable distribution of benefits within a halibut fishery that uses individual transferable quotas (ITQs) as a flaw in economic models which predict completely different behavior in ITQ-based fisheries. Sebele (2010) documents challenges to sustainably maintaining a community-based tourism destination in Botswana, as the community members feel the local elite use the Khama Rhino Sanctuary as their personal park. Iversen et al. (2006) document elite capture of forest user groups in Nepal, leading to structural instability. Other cases studies document the capture of community-based natural resource management by local elite, such as the forests in Cameroon (Brown & Lassioe 2009), communal farming in South Africa (Lebert & Rohde 2007), and agricultural land management in Australia (Pero & Smith 2008). Capture of community-based management by local elites is far from the exception (Platteau & Gaspard 2003), but is often not documented by researchers, perhaps due to pressure to publish only positive results (Mansuri & Rao 2004).

Equity and equality are related but not the same. Equality suggests the same share, while equity is concerned about the fair share. Baland and Platteau (1999) and Pérez-Cirera and Lovett (2006) both cite Mansur Olson's *The Logic of Collective Action: Public goods and the theory of groups* (1965) as a counter theory to suggestions that community-based management must distribute benefits equally (as opposed to equitably). Olson suggests that minor inequality increases incentives for powerful interests to discourage free-riding by those who would gain less by contributing, while major inequality works against collective outcomes. Dasgupta and Beard (2007) offer support for this conjecture in community-driven economic development programs in Indonesia, where they state that "(i)n cases where the project was controlled by elites, benefits continued to be delivered to the poor, and where power was the most evenly distributed, resource allocation to the poor was restricted" (Dasgupta & Beard 2007:229).

As a means of quantifying the level of equality, as a component of equity, this paper suggests using the Lorenz Curve and the Gini coefficient, two analytical methods for evaluating the distribution of an attribute among a population.

3.2 Quantifying Equality: Deriving the Lorenz Curve and Gini Coefficient

The "Lorenz Curve" is named after Max O. Lorenz, who developed a method to graphically represent the concentration of wealth within a population (Lorenz 1905). This method orders equal-sized segments by the amount of wealth each segment has, such that a cumulative total is obtained with the addition of each segment. For example, populations are often segmented by quintiles (fifths), and a hypothetical five segments might have the following percentages of the total wealth: 4%, 10%, 15%, 21%, and 50%. This is a deviation from a uniform distribution in which each quintile has 20% of the wealth. The two lowest segments cumulatively account for 14% of the wealth, and the four lowest segments cumulatively account for 50% of the wealth. Graphically, Figure 3.2.1 shows the curve generated by plotting these points. The Lorenz Curve is the curve formed by the hypothetical distribution posited above. Additionally, the "Line of Equality" is formed by the uniform distribution.

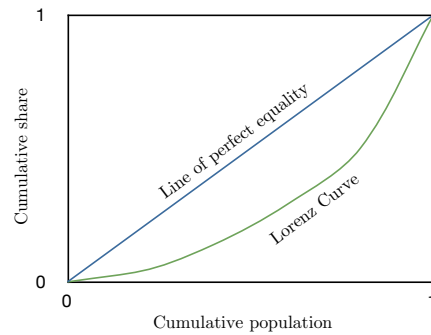


Figure 3.2.1: Lorenz Curve

A common measure of inequality is to examine the area between the Line of Equality and the Lorenz Curve (Area A in Figure 3.2.2, representing the deviation from equality), and take its ratio to the total overall area (Areas A+B in Figure 3.2.2). This ratio is known as the Gini Coefficient (Sen & Foster 1997:30). Gini coefficients range from 0 (no deviation from equality) to 1 (complete deviation from equality).

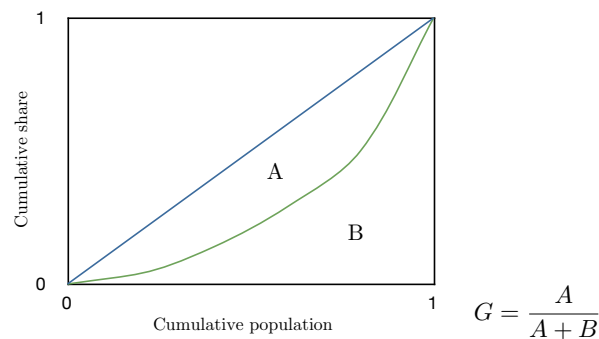


Figure 3.2.2: Gini Coefficient from Lorenz Curve

Initially the method for calculating the Gini coefficient was proposed differently by its originator Corrado Gini, using a method known as "relative mean differences" (Dalton 1920). Dalton (1920) attributes Umberto Ricci in *L'indice di variabilita e la curve dei redditi* (1916) as the first to publish a proof connecting the Lorenz Curve with the Gini coefficient, although Dalton says potentially Gini himself offered the proof. Sen and Foster (1997:31) give the formula in Equation (3.2.1) for calculating the Gini coefficient when all data points are known and unordered and also discuss early analysis of the Gini coefficient.

$$(3.2.1) \quad G = \frac{\sum_{i=1}^n \sum_{j=1}^n |x_i - x_j|}{2n^2\mu}$$

The absolute difference between each pair of data ($|x_i - x_j|$, where x_i and x_j are members in the set being compared) is calculated and summed (\sum) for each member; this quantity is divided by twice the number of data points (n) multiplied by the data set mean (μ). A fundamental aspect

is that the Gini coefficient is one half of the relative mean difference between all pairs of data (Dalton 1920, Sen & Foster 1997:30).

This formula can be easily programmed into a scripting language such as PHP (<http://www.php.net>) for quick calculation of Gini coefficients. Figure 3.2.3 gives a listing of PHP code that will calculate the Gini coefficient for a set of values manually entered into an array. Using the hypothetical data discussed earlier and the above PHP code, the Gini coefficient for a distribution of quintiles of 4%, 10%, 15%, 21% and 50% is 0.51.

```
<?php
$av = 0.0;
$G = 0.0;
$p = 0.0;

$ar = array(0.04, 0.10, 0.15, 0.21, 0.5);

$c = count($ar);
for ($i = 0; $i < $c; $i++) {
    for ($j = 0; $j < $c; $j++) {
        $a = abs($ar[$i] - $ar[$j]);
        $sum = $sum + $a;
    }
    $t = $t + $ar[$i];
}
$av = $t/$c;
$G = $sum/(2 * $c * $c * $av);
$cG = $G * ($c/($c-1));
print("Gini: $cG\n");
?>

$php -f Gini.php
Gini: 0.515
```

Figure 3.2.3: Sample PHP code to calculate Gini coefficient

Alternatively, if the mathematical function for the Lorenz Curve is known, the area under the curve can be calculated using integration over the function between the limits 0 and 1 (representing from 0% to 100% of the population). This can be constructed with the recognition that the areas represented by A and B in Figure 3.2.2 total one half of the area in the unit square bounded by the two axis.

$$(3.2.2) \quad G = \frac{A}{A+B}$$

$$(3.2.3) \quad A+B = \frac{1}{2}$$

$$(3.2.4) \quad A = \frac{1}{2} - B$$

$$(3.2.5) \quad G = \frac{A}{A+B} = \frac{\frac{1}{2} - B}{\frac{1}{2} - B + B} = \frac{\frac{1}{2} - B}{\frac{1}{2}} = 2\left(\frac{1}{2} - B\right) = 1 - 2B$$

$$(3.2.6) \quad G = 1 - 2B$$

Let $L(x)$ represent the mathematical function generating the Lorenz Curve, with limits from 0 to 1:

$$(3.2.7) \quad B = \int_0^1 L(x)dx$$

$$(3.2.8) \quad G = 1 - 2 \int_0^1 L(x)dx$$

There are variations of deriving the Gini coefficient, but the three equivalent methods above frame the fundamentals. In some cases, the Gini coefficient may be multiplied by 100. In such cases, a Gini coefficient of 37 is the same as 0.37.

3.3 Criticism of Gini coefficient

Criticism of Gini's 1912 book *Variabilità e mutabilità* occurred almost immediately upon publication. In 1913, the anonymous reviewer "E.C.S." took issue with both Gini's examples and the lack of "probable error or any indication of the range of variations" in Gini's development of indices, and ended with "To the statistician who proceeds systematically by the method of moments they offer nothing of great interest (ECS 1913:327)." This complaint seems misplaced. The Gini coefficient does not speculate or offer probability; as Gini indicated in his response (Gini 1913), it is merely a measurement of the distribution of data within a set, as the data was observed.

In cases where population members are grouped, the greater the granularity of the data set being analyzed the higher the Gini coefficient (meaning more unequal distribution). This is due to averaging within groups. The method of finding the ratio of the area of deviation away from equality to the total area is susceptible to this flaw. Sen and Foster (1997) suggest using the direct calculation:

Undoubtedly one appeal of the Gini coefficient, or of the relative mean difference, lies in the fact that it is a very direct measure of income difference, taking note of differences between *every* pair of incomes (Sen and Foster 1997:31) (emphasis in original).

The Lorenz Curve does have an advantage over the Gini coefficient in terms of visually presenting

information, including areas of flatness or sharp increases, representing no additional increase or a significant increase, respectively, in the percentage of the resource accounted for by the addition of another population segment. This information is lost when calculating the Gini coefficient. There have been multiple enhancements to methods to calculating the Gini coefficient and one such method is to use decomposition to obtain individual components of inequality (see Lerman & Yitzhaki (1985), e.g., which will be reintroduced, briefly, later in this chapter).

The Lorenz Curve and Gini coefficients do not offer insight into the population demographics. For example, in a Lorenz Curve showing the distribution of wealth in a nation, there is no distinction between young and old members, who are likely to have substantially different levels of acquired savings (Paglin 1975). Last, the Gini coefficient makes no statement regarding the difference between equality and equity. This has been noted by several authors in their analyses (see Paglin (1975), Cullis and van Koppen (2007), Wang et al. (2007), e.g.). This will be revisited below.

In practice, the Gini Coefficient can not be 1. From Eqn. 3.2.1, the Gini Coefficient is calculated from the differences between each pair.

$$(3.2.1) \quad G = \frac{\sum_{i=1}^n \sum_{j=1}^n |x_i - x_j|}{2n^2\mu}$$

Consider a normalized distribution, where 100% of the resource goes to a single individual among a group of four individuals. Summing over one individual with 100% of the resource (x_1) yields the following:

$$(3.3.1) \quad \sum_{j=1}^n |x_i - x_j| = |x_1 - x_1| + |x_1 - x_2| + |x_1 - x_3| + |x_1 - x_4|$$

The first difference is zero; the other differences are equal to 1, so this summation equals $(n-1)$.

$$(3.3.2) \quad \sum_{j=1}^n |x_i - x_j| = \underbrace{|x_1 - x_1|}_0 + \underbrace{|x_1 - x_2|}_1 + \underbrace{|x_1 - x_3|}_1 + \underbrace{|x_1 - x_4|}_1 = n - 1$$

This is repeated for the summation of the remaining group members. These members have 0% of the resource, so their differences with each other and themselves is zero. Only the difference between that group member and the group member with 100% of the resource have non-zero values, in this case 1:

$$(3.3.3) \quad \sum_{i=1}^n |x_i - x_j| = \underbrace{|x_2 - x_1|}_1 + \underbrace{|x_3 - x_1|}_1 + \underbrace{|x_4 - x_1|}_1 = n - 1$$

The summation over all pairs yields:

$$(3.3.4) \quad G = \frac{\sum_{i=1}^n \sum_{j=1}^n |x_i - x_j|}{2n^2\mu} = \frac{2(n-1)}{2n^2\mu}$$

The top and bottom 2s cancel. Noting that in the case of all of the resource going to a single individual, the mean μ is $1/n$,

$$(3.3.5) \quad G = \frac{2(n-1)}{2n^2\mu} = \frac{n-1}{n^2\frac{1}{n}} = \frac{n-1}{n}$$

This result, $(n-1)/n$, will always be less than 1. Eqn. 3.2.13 represents the maximum possible value for the Gini Coefficient in a group ($G = (n-1)/n$). In a simple cases such as $n = 2$, all of the resource going to one of the two individuals yields a Gini Coefficient of 0.5, even though there is perfect inequality in distribution. As such, Gini Coefficients approaching 1 can only be achieved with large numbers in a set with a high degree of concentration among only a few members. To remove bias from the set, the Gini Coefficient must be multiplied by the correction factor $n/(n-1)$ for small sets. Dixon, Weiner, Mitchell-Olds, and Woodley (1987) suggest $n < 100$ is an appropriate guideline, although the bias exists in all uncorrected sets. The PHP code in Figure 3.2.3 contains this correction.

The converse is not true, however. In a distribution in which the resource is distributed completely evenly, the differences of each pair is always 0. Therefore, the lowest possible Gini Coefficient is 0.

3.4 Using the Gini coefficient

The most common usage of the Gini coefficient is in economics, to measure income equality (or the lack thereof). One example is that of Canadian income and wealth distribution, in comparison to the United States of America (USA). Canada's pre-tax income distribution in 2005 was 0.32, while the USA was .45 in 2007 (CIA n.d.). Canada's net worth Gini coefficient was .659 in 2005 (Brzozowski et al. 2010), whereas it was .77 in 2006 in the USA (Heathcote, Perri & Violante 2010). Conceptually this equates to the top 20% controlling 63.1% of the wealth in Canada (Davies et al. 2011) and 84% of the total wealth in the USA (Ariely & Norton 2011). In both countries, the bottom 20% have a negative net worth (Davies et al. 2011). An example of group averaging creating an artificially lower Gini coefficient is that while 84% of the total wealth in the USA is owned by the top 20% of the population, estimates put 57.7% of the total wealth in the hands of only 5% of the population, with 32.7% in the top 1% (Davies et al. 2011). Averaging the top 5% and 1% within the top 20% masks the true level of concentration and lowers the Gini coefficient.

An advantage of using the Gini coefficient for comparisons is that it is independent of the scale of the attribute being measured. While the USA, Iran and Mozambique have substantially different economies in terms of size and GDP, they can be compared against each other in terms of income inequality (all three have pre-tax income Gini coefficients of .45 (CIA n.d.)).

Wilkinson and Pickett (2009) took this approach and analyzed the outcomes of a large number of social ills across many countries, in their book *The Spirit Level: Why Equality is Better for Everyone*. With only the occasional exception, Wilkinson and Pickett found a strong correlation between inequality within a country and unfavorable outcomes for social ills such as teen preg-

nancy, alcoholism, crime and incarceration rates, and obesity. Taking this approach to the community level, Modrek and Ahern (2011) apply the Gini coefficient to "cantons" in highly homogenous Costa Rica and find some support for decreased health within communities with unequal wealth and income distributions. Some factors that could not be controlled for included migration within the country and the long period before onset of diseases associated with inequality. Nonetheless, Modrek and Ahern concluded that inequality at a local level is likely to contribute to less favorable outcomes in health issues.

This is not to suggest that the matter is truly settled at the local level. Potentially there is a minimum level of inequality that must be crossed before there is an impact. Using the Gini coefficient, Kravdal (2008) found little support for the income inequality hypothesis in Norway, a country with one of the lowest Gini coefficients in the world (CIA n.d.). Zhang, et al. (2011) used a "deprivation inequality" as a measure of affluence between neighboring areas in England, and found support for Wilkerson and Pickett's income inequality hypothesis but found a higher mortality rate among the more affluent within disparate neighborhoods, in spite of generally accepted views that mortality rates are lower among the more affluent members of society. Auger, Zang, and Daniel (2009) cite an inability to calculate the Gini coefficient from available data as an impediment to quantifying inequality in communities in Québec, Canada, given that the other indicators they used (decile ratio, coefficient of variation, and mean share) showed an inverse relationship between income inequality and mortality rates (due to alcohol, tobacco and suicide). Stafford et al. (2005) note that differing indicators of inequality have led to differing conclusions in health outcomes. Fundamentally, though, community-based management should concern itself with benefit distributions locally.

Other uses of the Gini coefficient can be found in literature on natural resource management, including the management of common pool resources. In a comprehensive application of the Gini coefficient at a global scale, White (2007) uses the Gini coefficient, Concentration Curve and the Atkinson Index to analyze global resource consumption. The data is drawn from the Ecological Footprint (EF), which is stated as the "amount of bioproductive land necessary to support a given level of consumption." The EF allows direct and indirect uses of resources to be captured by a single measure (land) but is not without controversy. White (2007:405) further deconstructs the composition of the EF into four separate land uses: Energy ("land for fuel wood, fossil fuels and nuclear energy"), Forest ("timber, pulp wood and paper"), Food ("grazing land, crop land, and fishing grounds") and Built (infrastructure), based on the World Wildlife Federation International National Footprint Accounts (WWF International 2006). This allows disparate contributions to the overall Gini coefficient to be identified. Data from 140 counties was analyzed to generate the four individual GCs and the Total Footprint GC. The author utilizes a variation of Eqn. 3.2.6 above, by noting that it can be solved such that G is equal to twice the area between the line of equality and the Lorenz Curve (area A in Figure 3.2).

White (2007:405) found that "Forest footprint" ($G = 0.663$) had the largest inequality among the 140 countries, meaning that a few countries consume the largest portions of land used glob-

ally for timber products. "Energy footprint" ($G = 0.553$) had the second largest inequality, while the "Built footprint" ($G = 0.390$), and the "Food footprint" ($G = 0.272$), were third and fourth, respectively. The Total Gini coefficient was 0.446, which is not the average of the four subcategory Gini coefficients. Each country may have substantial differences between two subcategories, but the countries can have the same Gini coefficient if one subcategory offsets another one within that country. The order of the subcategories for the countries will be different, leading to a unique Total Gini coefficient. Additionally, each subcategory contributes to the total land use differently, and the results need to be weighted. To deal with this, the author introduces the Concentration Curve, which orders subcategories "according to the variable of interest (Total Footprint, in this case)." This correction shows that Energy contributes 65.6% of the total inequality, with Food (20.1%), Forest (11.2%) and Built (3.2%) following. White concludes "...the methods used in this paper to describe global inequality could easily be applied to the distribution of resource use on a national or local level provided the availability of adequate data" (White 2007:409).

Chakraborty (2001) uses the Gini coefficient to characterize the land distribution in Nepal while analyzing the outcomes of common pool forestry management institutions, and identifies these management institutions as responsible for the distribution of access to forest products. Chakraborty (2001) identifies issues with inter- and intra-group inequities in access and distribution of benefits as challenges to sustainable management of the forest commons, but does not apply the Gini coefficient to that distribution. Fum and Hodler (2010) find income inequality among natural resource rich countries increases with a few large "polarized" ethnic groups, but decreases in countries with many small ethnic groups. To this end, Pérez-Cirera and Lovett (2006) construct a model using Gini to inform government authorities which community forests (ejidos) need greater oversight due to power imbalances.

Lerman and Yitzhaki (1985) developed a method for determining which component contributes the most to inequality, in cases where there are multiple income sources, and decomposition is possible. This is used by Babulo et al. (2009) to isolate forest products as contributions to income in rural Tigray, Ethiopia, finding that access to forest products reduces poverty and inequality and as such should be incorporated into forest management plans. Similarly, Mamo, Sjaastad, and Vedeld (2007), in studying income inequality in Dendi, Ethiopia, constructed two Gini coefficients, one including forest products-dependent income and one not, to isolate the contributions from access to forest products. They also found a reduction in income inequality from access to forest products. Kant, Nautiyal, and Berry (1996) are even more granular, looking only at non-timber forest products, and conclude the same, that inclusion of these products decreases income inequality

National and local level programs such as "social assistance, unemployment benefits, and various child benefit programs" (Brzozowski, Gervais, Klein, & Suzuki 2010:53) are important distributive policies whose presence reduces inequality leading to lower Gini coefficients; the absence of such could increase inequality and the Gini coefficient (Brzozowski, et al. 2010). Additionally,

long periods of prosperity (Brzozowski, et al. 2010) or rapid growth (Scully 1991) increase inequity in economies; conversely, recessions work towards reducing inequity (Brzozowski, et al. 2010).

Outcomes of government policies can be quantified using the Gini coefficient. Lee (2009) examined income in tourism services-dependent communities in the U.S. and found that income inequality increased in all of them between 1990 and 2000. The highest increase in inequality came from mountain ski resorts, mirroring the ongoing challenges in Whistler, B.C., where services employees have difficulties finding affordable housing in Whistler (Gill & Williams 2011). The "hypothetical" distribution earlier, where the top 20% own 50% of the wealth, came from Lee (2009), as the wealth distribution in a typical mountain ski resort county. In contrast, Lee (2009) found counties with national parks had the least increase in inequality, and counties dependent on manufacturing had no changes in inequality. These findings highlight the need to examine income distributions in efforts to attract tourism to communities.

The Gini coefficient can be used to examine the effects of corruption on economic development. Gupta, Davoodi, and Alonso-Terme (2002) found a clear correlation between government corruption and unequal income distributions, with greater levels of corruption also impeding economic development at the lower income levels. The reverse inference is likely true; high levels of unequal income distribution are suggestive of government taxation and allocation policies and practices that favor special interest groups having potentially immorally- (through anti-democratic laws) and/or illegally- (through bribes) gained influence. An example of ostensibly immoral (but legal) influence can be found in Cullis and van Koppen (2007) review of water licenses on the Oliphants River in South Africa. In spite of the end of apartheid, Cullis and van Koppen found the distribution of water allocations to have a Gini coefficient of 0.96, where 0.5% of the population controlled 95% of the water, with race playing a significant role.

As a last example, Wang et al. (2007), in a study on the distribution of water supply and demand on the Yellow River in China, developed an integration method as described above based on a step-wise population function of water consumption, instead of income. The authors found that inequality in water consumption had peaked in 2001 and dropped in the following five years, and further investigation was warranted into balancing equality and equity in terms of economic activity.

3.5 Identified gap: Distributing benefits using the Gini coefficient as a policy guideline

While there is a wide range of literature that includes the use of the Gini coefficient as a measurement of the distribution *after* the distribution has occurred, there appears to be a shortage of literature discussing using the Gini coefficient *in advance* of the distribution. This author has been unable to locate any case studies in which a community-based management institution used the Gini coefficient as a guideline for distributing benefits. This paper will now develop a

method for applying the Gini coefficient to the distribution of benefits, in conjunction with the average path length per vendor developed in Chapter 2, to provide a quantifiable approach to distributing benefits from community-based management of a natural resource. This is done through balancing the average path length of resource consumers with a distribution that is at least as equal as that given by a particular Gini coefficient. As will be shown, it is not possible to simultaneously maximize both the average circulation of currency and the distribution of benefits. Rather, an optimal trade-off must be found.

Benefits may range from harvesting and extraction opportunities to the disbursement of funds through grants to community groups. As noted by Pinkerton et al. (2008), access to the timber by local mills is a benefit of community-based management of forests. For this example, in using the Gini coefficient as a policy guideline, a hypothetical community with local supply chains will be used. These supply chains begin with businesses that purchase timber from a community forest, and who then sell it to a local mill. These local mills may sell the wood to local value-added manufacturers, which may then sell to retailers. The specifics of the supply chain will not be documented, and the example supply chains will be simplified, by considering only sequential transactions, and exaggerated, by positing unrealistically high lengths, for the purposes of illumination. Chapter 2 explained how this supply chain represents the average path length of currency from the time it enters a community to the time it leaves. The resource itself flows in the opposite direction from the currency.

The supply chain also represents effort towards improving the community's capture of benefits from the natural resource, which is a component of one of Ostrom's design principles, that of proportional equivalence between benefits and costs (Ostrom 1992). Conceivably the business, as well as other components on the supply chain, has to make concessions towards sharing costs in order to achieve long supply chains locally. Reasonably, then, these longer supply chains can expect to receive a greater proportion of the benefits from the natural resource. At the other end of the spectrum, the business with the shortest average path length benefits the community the least, and should expect to receive the smallest share.

For this example, four average path lengths will be used: 8.53, 5.23, 2.98, and 1.0, from businesses owned by Alexandra, Bob, Carl and Doug, respectively (See Figure 3.5.1). For illustrative purposes, assume that these businesses are the only businesses in the supply chain that can utilize timber directly purchased from the community forest. The first (Alexandra) represents a value-added business who has achieved complete processing of the timber locally, while the last (Doug) represents a timber buyer who is from out-of-town and uses non-local labor while selling the timber out-of-town as well. Conceivably this timber buyer also offers the highest price for the timber, or some other incentive as a reason to be included in the distribution of access to the timber.

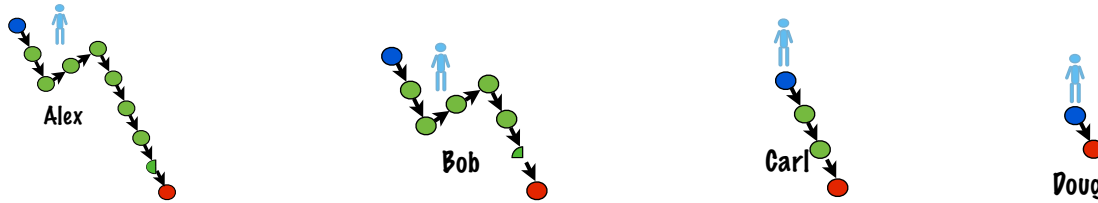


Figure 3.5.1: Graphic depiction of business path lengths

If the community chooses to maximize local economic activity, all of the timber would go to Alexandra, who has the longest local supply chain. Using Eqn. (3.2.1), the Gini coefficient of this arrangement is 0.75, the maximum possible Gini coefficient for this number of members in a set:

$$(3.5.1) \quad G = \frac{n-1}{n} = \frac{4-1}{4} = \frac{3}{4} = 0.75$$

Corrected for bias (multiplying by $n/(n-1)$, or $4/3$), this Gini coefficient is 1.0. If the community chooses to maximize income received from sales of the timber, all of the timber would go to Doug, who offers the highest price, but again this has an uncorrected Gini coefficient of 0.75. If the community chooses to maximize the distribution of access to the timber, the Gini coefficient is 0.0 (no differences in pairs, so the summations equal 0). The projected circulation of currency within the logging community is then the weighted average of each business's supply chain. With four businesses and uniform distribution, each business would get 25% of the access, or in this specific type of example, 25% of the total allowable harvesting of timber for a given period. The aggregate average path length (L) is then the sum of 25% of each business's average path length.

$$(3.5.2) \quad L = .25 * 8.53 + .25 * 5.23 + .25 * 2.98 + .25 * 1.0 = 2.13 + 1.3 + .75 + .25 = \mathbf{4.435}$$

This aggregate average path length L of 4.435 is less than the maximum possible of 8.53, obtainable by allocating all of the harvest to Alexandra. Maximizing one variable, such as the distribution of access or the maximum economic activity or return, comes at the expense of the other variables. Therefore, the goal should be to find the optimal balance of variables. For this example, the balance will be between the collective community economic activity and the distribution of access, while leaving out economic return for the community institution managing the resource (basically assuming a fixed price for timber).

As discussed earlier in this chapter, small imbalances in power and benefits may be beneficial to community-based management. Attempting to achieve perfect equality in the distribution of benefits may be counterproductive. However, excessive imbalances in power and benefits are likely to lead to conflict and eventually to unsustainable utilization of the natural resource and/or capture of control of the resource. Therefore, in managing the resource, a reasonable goal is to choose a distribution that is reflective of a wider community profile. For example, as noted earlier, the pre-tax income distribution Gini coefficient in Canada is 0.31. The following exam-

ple will use a corrected 0.35 as a maximum allowable inequality in the distribution of benefits. However, in practice, communities may choose a lower Gini coefficient, particularly in cases where there is minimal differences in average path lengths among resource consumers.

If Alexandra receives 50% of the allocation, and each of the other three businesses receive 1/3 of the remaining 1/2 of the allocation, the corrected Gini coefficient is 0.33 (as calculated by the PHP script in Figure 3.2.3) and the aggregate average path length L is 5.8.

$$(3.5.3) \quad L = .5 * 8.53 + .1667 * 5.23 + .1667 * 2.98 + .1667 * 1.0 = \mathbf{5.80}$$

This Gini coefficient is below the guideline of 0.35, but already shows a high concentration towards the top 25%. Communities with many businesses may find it easier to obtain low Gini values (indicating a tendency towards equal distribution), but where communities have only a few businesses, there will be a difficulty in not concentrating timber sales. The burden should then be on the businesses to ensure their returned effort is proportional to this extra benefit.

Tweaking of the Gini coefficient for a longer average path shows that a Gini coefficient of 0.416 (Alexandra receives 50%, Bob receives 25%, and Carl and Doug each receive 12.5%) has an aggregate average path length of 6.07. An additional allocation change, where Alexandra receives 47%, Bob receives 23%, Carl receives 17% and Doug receives 13%, yields a Gini coefficient of 0.346 and an aggregate average path length of 5.86. This appears to be the optimal balance, and further calculations are unlikely to offer substantial increases in the aggregate average path length. To increase the aggregate average path length, more allocation must be given to Alexandra, but doing so will exceed the upper limit of the Gini coefficient. As discussed in the criticism of the Gini coefficient, other distributions may yield the same Gini coefficient. Attempting to find which distribution has the highest aggregate average path length among identical Gini coefficients would be instructive towards which one to choose. Surprisingly, the initial allocation was close in both Gini coefficient and average path length, although this does not reward Bob and Carl for their longer supply chains.

3.6 Chapter Summary

Issues of power and benefit imbalances are important in community-based management. Having an ability to quantify these imbalances is very helpful in monitoring and addressing inequities. There is some evidence to support the hypothesis that small imbalances create greater cooperation towards collective action, as those who benefit more are likely to encourage those who stand to gain less, but large imbalances lead to strife and capture of the institution managing the resource. These same imbalances may lead to local inequalities with possible impacts on health and other social indicators of well-being.

The Gini coefficient is an easy-to-use measurement of unequal distributions of benefits from the community-based management of natural resources. It is flexible enough to evaluate concerns

about health outcomes, corruption, and inequity in government policies, while providing a mechanism to guide communities about the distribution of benefits. To that end, an example was provided of how the Gini coefficient can be used to allocate harvesting opportunities to local businesses. Communities are free to decide what are appropriate levels of unequal distribution of benefits, while also encouraging greater responsiveness to community needs in order to obtain larger shares of the benefits.

This analysis only considered the head of the supply chains as being members of the group receiving benefits. Clearly the downstream businesses do as well. The economic mapping discussed in Chapter 2 can show where supply chains may cross at the same business. Under such circumstances, this business, which would have a high "betweenness centrality," may benefit more than other businesses in the same profession. Further analysis examining the integration of the Gini coefficient, average path lengths, and graph theory metrics may offer tools to extend this quantification of equality in the distribution of benefits.

4. Case Study: Dunster, British Columbia

4.1 Background

The Robson Valley area of central eastern British Columbia is formed by two mountain chains coming together, the Caribou Mountains from the southwest and the Columbia Mountains (part of the Rocky Mountains) from the northeast. These two mountain ranges frame a narrow 12 kilometer wide corridor running approximately 300 kilometers between Prince George to the west and Valemount to the east. The headwaters of the Fraser River form at the eastern edge of the Robson Valley, and the river flows through the Robson Valley northwest to Prince George before turning south through interior B.C.

Robson Valley's geography creates a unique geoclimatic condition that leads to the world's only wet and cool (classification wk3), and very wet and cool (classification vk2) "inland temperate rainforest" (Goward & Spribille 2005). The upslope of air currents against the mountains on either side of the Robson Valley leads to a high level of precipitation (usually snow) in the wintertime, with long intervals between stand-destroying disturbance events (Sanborn et al. 2006).

Near the center of the Robson Valley is a small, unincorporated area called Dunster. The community has a community forest agreement with the provincial government of British Columbia. One incorporated municipality and several small unincorporated communities currently exist in Robson Valley, while several others have periodically been occupied and then abandoned in the last 100 years (Wheeler 1979:1-18). Stamm (2004) reports between 80 to 85 households in the Dunster area, representing approximately ten percent of the 917 households in the Fraser-Fort George H Area (Statistics Canada, 2007). An estimation of fewer than 180 people in the Dunster and surrounding areas is likely to be an acceptable figure to Dunster residents. The entire population of Robson Valley is approximately 2000 people (Statistics Canada, 2007b).

4.2 Dunster Community Forest Society as partner

As part of the FFESC grant, the Dunster Community Forest Society participated in the surveying of the local community economy. Dunster as a location provides few direct jobs. With the closing of the Dunster Fine Arts School at the end of the 2009-2010 school year, multiple-person employment has centered on agriculture and small mills employing one or two persons, plus some individual logging and forestry efforts. A General Store provides employment for a few individuals as owners of the store. Carrier Lumber provides out-of-town employment for an unknown number of other individuals.

In spite of this, or perhaps because of it, Dunster exhibits a very tight-knit community spirit. Much of the funding for the Community Hall and efforts to re-open the School comes from community fundraisers. The annual Robson Valley Music Festival in Dunster attracts a wide range of musical acts and tourists for three days in August. Additionally, the Dunster Ice

Cream Social in June is a widely-attended activity, when hundreds of people will pay an entrance fee for unlimited ice cream and pie.

Dunster has pursued a community forest agreement since 2002 (A. McLean, pers. comm. 2011). One obstacle the Dunster Community Forest Society (DCFS) has faced is that it represents an unincorporated township. This has created both financial and legal obstacles to be overcome. After several years of lobbying, DCFS secured an invitation to apply from the Ministry of Forest and Range, and in December of 2009, DCFS was awarded a 25 year Community Forest Agreement. The land tenure consists of 20,000 hectares and a 15,000m³ Annual Allowable Cut (AAC) (DCFS 2007). The tenure is on both sides of Robson Valley, but made non-contiguous by private property on both sides between the tenure areas and Highway 16. Much of the tenure is on the south side of the leading edge of the Caribou Mountains, in the Raush River Valley. This area is accessible through a 22 km Forest Service road.

The land for the tenure has been logged by Carrier, a veneer plant in the Robson Valley, and another community forest, under its previous salvage license. While a Timber Supply Analysis was done for the tenure, there has been no ability to verify the accuracy of the data. Interviewees unanimously expressed frustration over not knowing what timber is still left. The general consensus is that what is left is of low economic value and in difficult logging locations. The timber profile is largely spruce and pine, with some areas heavily populated by pine, but also with significant numbers of mountain pine beetle (MPB) killed trees. Visual observation by the author showed several areas with significant browning characteristic of dead pine. The Dunster Timber Supply Analysis indicates a small amount of Interior Cedar-Hemlock ICH exists on the Raush Valley side of the tenure area (DCFS 2007). At the time of partnering with DCFS in June of 2011, harvesting had not yet begun, due to delays in receiving a cutting permit for their tenure area, under a new "single cutting permit" program in the Ministry of Forests, Land and Natural Resource Operations.

DCFS has encouraged and sought out a relationship with the Simpcw First Nations from early in the community forest application process. Some disagreement exists as to whether this relationship was initiated due to a recognition of a land claim or due to a need to build legitimacy in which DCFS gains Simpcw support while the Simpcw gain a Director's seat on DCFS. Regardless of the origins of that seat, there is unanimous agreement that the relationship with the Simpcw has been very beneficial to DCFS efforts. A common statement among interviewees was the desire for the Simpcw to be more involved, as the distance to the meetings generally required that the Simpcw representative be present only by phone and not in person. DCFS recognizes that Simpcw maintained a residency at the headwater of the Raush River, which is contained within the DCFS tenure footprint (R. Howard, pers. comm., 2011).

4.3 Identification of businesses in Dunster

Due to a change in partners coming after an ethics review had been completed, a narrow mecha-

nism for identifying businesses was adopted. Prior to the start of field research, the SFU Office of Research Ethics had expressed concern about the author's methods of identifying persons to contact. At issue is the concept of "snowballing," where a contacted person indicates other persons to contact, without obtaining permission from the other persons to reveal their identity. This approach was not condoned by the SFU Office of Research Ethics.

The model developed to measure the circulation of currency focused exclusively on expenses from businesses to other businesses. Unlike individuals, businesses do not have the same level of assumed privacy. Businesses and individuals are free to identify businesses they spend money at, without requiring permission from the business first. However, in Dunster, many individuals have "businesses" by which they obtain income, but not all of these are publicly identified as such. In order to avoid ethical issues caused by snowballing, businesses were only included if they had "self-identified" as such, through web pages, newspaper advertisements, phone book listings, or business cards posted on bulletin boards. If a business identified an individual as a recipient of expenses, and that person could not be identified as a business, that person was listed as an anonymous business and tracked internally. Thirty businesses were identified in Dunster.

4.4 Surveys

To obtain the necessary information, the author developed a survey with the cooperation of Archie McLean, the Chair of the Dunster Community Forest Society. The survey requested the listing of expenses by percentage to businesses for the years 2007, the year before the community forest agreement was awarded, and 2010, the most recent tax year. Examples of how to fill out the survey were provided in the survey. Appendix C is the survey.

The surveys were preprinted and enclosed in a stamped envelope pre-addressed to the author's mailbox at the location he was staying at for the summer. The author constructed a small, open box with a sign attached to it, to hold the surveys. On July 14, 2011, this box was first placed on the counter at the Dunster General Store, a central location in Dunster that contained the mailboxes for the community. The box remained on the counter except on Saturday mornings, as will be discussed below, until August 25, 2011. As an admittedly "tongue in cheek" gesture, the sign indicated that submitted survey owners were entitled to a free cookie, which the author had pre-purchased and left with the front register of the Dunster General Store. During the survey period, 49 surveys were printed and distributed through the box or direct interviews. Of these, 13 surveys were returned, 12 of which were useable (the remaining was from a business outside of the Dunster area). Four of the surveys were returned by mail, six were filled out in person, and three were sent by email after August 25, which were then manually entered into a paper survey. Two surveys were recovered in an incomplete form, and five surveys were recovered from the box at the Dunster General Store. Twenty-nine surveys were unaccounted for.

Status	#
Printed	49
Returned	13
Useable	12 (40% of 30)
Done in person	6
Returned by mail	4
Returned by email	2
Recovered from box	5
Unaccounted for	29 (59.1%)

Table 4.4.1: Status of surveys

4.5 Actions in support of encouraging participation

In addition to the support and effort on behalf of the survey by the Dunster Community Forest Society, the author undertook several actions to generate interest in the survey. One individual in the community maintains an electronic mailing list to post messages. This electronic mailing list goes out to approximately 130 residents of the Dunster area (P. Amyoony, pers. comm.). The author posted to this mailing list on July 15, July 21, July 28, August 12, August 20, and August 21, 2011. The postings explained the purpose of the economic mapping project, informed the community of the author's presence at the weekly Dunster Farmer's Market, and as the five week period neared an ending, explained that businesses that had been identified but had not participated would be shown as having all of their expenses leaving the community.

The Dunster Farmer's Market is a significant gathering place on Saturday mornings during the summer months. On July 23, 2011, the author volunteered to flip burgers to help raise funds for the Dunster Fine Arts School. Over the course of two hours, the author grilled 47 hamburgers, which sold for \$5.50 each, and five hotdogs, which sold for \$3.50 each. On July 30, August 6, and August 13, 2011, the author operated a booth to meet with prospective business owners. During this time the box containing the surveys was removed from the Dunster General Store and was at the Dunster Farmer's Market with the author. The booth displayed a sign identifying the booth as part of the economic mapping survey and a sample map. During these days the author met with over 15 community members to explain the survey and economic mapping project. Two surveys were completed during this time. Several individuals indicated they wished that the survey included personal expenses, as they wanted to participate but were excluded. On August 20, 2011, the author again grilled hamburgers, although due to competition from the Robson Valley Music Festival a much smaller number of burgers were sold. However,

almost twice as many hotdogs were sold, as the author gained a positive reputation for being able to properly burn hotdogs evenly (hotdogs being best when crisp on the outside).

During the six-week period for which the survey was open, the author interviewed a total of seventeen Dunster area business owners, including those spoken with at the Dunster Farmer's Market. The author located phone information for other Dunster area businesses and "cold called" thirteen additional businesses. In total, twenty-nine "businesses" were identified in the Dunster area, and contact was made with twenty-eight of them.

On July 17, 2011, the author participated in a "workbee" day to help clean up the Dunster Fine Arts School, and to obtain a more reliable water supply for the school from the nearby stream. On the same day, the author was interviewed by the editor of the Rocky Mountain Goat newspaper out of Valemount, B.C., on the economic mapping project. An article about the project with a picture of the author and a graphic showing the four node recirculation model discussed in Chapter II appeared in the July 27, 2011 issue of the Rocky Mountain Goat (RMG 2011a). On July 8 and July 18, the author and DCFS Chairman Archie McLean took exploratory trips into the Raush River Valley area, leading to the identification of three previously unknown Old Growth ICH patches within the DCFS tenure area. During the July 20 and August 17, 2011 DCFS Board of Directors meetings, the author took the minutes and later typed these up for distribution to the board members. On August 16, 2011, the author coordinated the construction of the subfloor to a sound booth being built for the Robson Valley Music Festival.

Last, during the month of August the author coordinated with the Future Forest Ecosystem Science Council grant research team and the Dunster Fine Arts School Society to hold a community-driven conference where the results of the FFESC research would be presented. On October 13 and 14, 2011, the "Carbon, Climate Change and Community Forests (C4F)" conference was held in the newly renamed Dunster Fine Arts School and Convention Centre. Thirty-eight individuals attended on the first day and twenty-five individuals attended on the second day. Over the two days there were fourteen presentations by thirteen individuals ranging from academicians and MoFLNRO employees to community members. Through the concession lunches over the two days, the Dunster Fine Arts School Society generated over \$700, and an estimated additional \$900 went to motels in a nearby community. Dunster does not have any restaurants, motels or operating bed and breakfasts.

In general, the author's efforts were met positively, although the final participation levels of 12 out of 30 were lower than had been hoped for. Only three businesses provided information for both 2007 and 2010. One aspect of the survey presented a regular problem that may have deterred participation. In the survey, the author asked for percentages of expenses by vendor. However, many businesses tracked their expenses by category instead of vendor. For users with computers and accounting software, this should not have presented a significant obstacle, and the author prepared and distributed instructions on how to sort expenses by vendor instead of category. Three surveys, two useable, were submitted after benefiting from these instructions.

In Dunster, many businesses still used pencil and paper ledgers to track expenses. Two businesses agreed to sit down with the author and extract the information from their ledger, one of which agreed to do this after talking with the author while the author was grilling hamburgers at the Dunster Farmer's Market. Two other surveys were received as a result of interactions with business owners at the Dunster Farmer's Market, confirming the author's belief that maintaining visibility within the community was essential for building sufficient trust to participate in the survey.

4.6 Negative influences on participation

There was not universal acceptance of the author's efforts. Some businesses declined to participate, citing reasons ranging from a dislike of surveys to "none of your business." Notable is the brief correspondence the author had with a prominent community business owner who strongly disagreed with the approach taken by the author. Appendix D contains the full correspondence between the community member and the author. The community business owner made several suggestions regarding qualitative rather than quantitative analysis of buying locally which did not fit the mathematical model that was developed in Chapter 2. Efforts to explain the project to the community business owner were unsuccessful, resulting in a comment from the community business owner that "maybe it will be a great thesis about how not to do it." The community business owner was able to anticipate that businesses were unlikely to be able to track expenses by vendor, and clearly ended the communication on a negative note. The author was not able to gauge how much influence the community business owner may have had with Dunster businesses, but the community business owner would have been in a position to talk to Dunster community members as they examined the survey box.

4.7 Incentives to participate, non-respondents and missing surveys

As mentioned earlier, the box containing the surveys at the Dunster General Store said participants were eligible to receive a free cookie. This was not a serious effort at providing an incentive to survey participants. Although the literature on survey participation shows a clear benefit to providing a financial incentive for participating (see Church 1993 for a review), the author made the decision to not do so. Survey participation literature has identified three primary motivations for participating in surveys: 1) altruism, 2) issues related to the survey itself, and 3) incentives offered to survey-takers (Singer & Couper 2008). The use of incentives includes the notion of reciprocity, where a participant is fulfilling a social contract based on the perception of having received gifts (Groves, Cialdini, & Couper 1992). Survey-related motivations include topics that are of interest to the participant (Groves et al. 2006). Goyder (1987) identifies the legitimacy within the community of the institution sponsoring the survey as a survey-related motivating factor.

Through the participation of activities important to the Dunster community, the author provided an intangible benefit to the community. By partnering with the Dunster Community For-

est Society, the author established the legitimacy of the survey. Last, the issue of spending locally is important to many people in small communities. Taken together, profiles of respondents and non-respondents can be surmised. A respondent was more likely to support community-based activities, including the community forest, and be interested in localization, while a non-respondent is likely to be the opposite, in that supporting the local community forest and localization is not a priority.

The author used accounting software from his previous business efforts to complete the survey in approximately one hour per fiscal year. Monetizing incentives for otherwise non-respondents to participate would likely approach \$50 or more per survey. Given the suggested profile of the non-respondents, this would likely mean paying an otherwise non-respondent \$50 to report their business spent little or no money locally. As discussed in Chapter 2, the developed model already posited that non-respondents would have all of their expenses leaving the community, so this additional expense would not be likely to yield any additional information. Only the percentage of the non-respondents' expenses paid to themselves as a Dunster resident would have any impact on the calculated average path length.

The author believes that very little more could have been done to improve the participation rate in the survey. The criteria for the survey is specific, and responses within the survey ranged from precise knowledge to little or no knowledge of the business' expenses. The author's efforts to participate in activities important to the community did result in additional responses to the survey. Efforts by DCFS on behalf of the economic mapping project also resulted in responses to the survey. Interestingly, although all of the DCFS Board of Directors members were identified as business owners, only four Directors submitted surveys. One was unable to due to accounting issues. Two others did not submit surveys, indicating a potential lack of "buy-in."

The absence of twenty-nine out of forty-nine surveys is strongly suggestive of a high level of community interest in the survey. Although seventeen of the thirty identified Dunster businesses were not surveyed, there is no reason to assume that they account for the majority of the missing surveys. Rather, these surveys were likely picked up by community members that did not own businesses, only to realize they were not targeted for the survey. Businesses that could not provide the data may account for a percent of the missing surveys. Unfulfilled commitments to complete the survey account for six of the missing surveys.

5. Code

The database structure and code for the economic mapping survey and analysis are in Appendix E. The database software postgresQL and the programming language PHP were used to facilitate analysis.

6. Results and Modeling

6.1 Social network analysis: Methodology

The collected data for the Dunster economy was formatted into the DL language (See Chapter 2) and exported into a file, which was then imported into the social network analysis software Gephi (<http://www.gephi.org>). This software was used to render a map of the Dunster economy (see Figure 6.1.1). The arrangement of the network was done manually, with an eye more towards aesthetics than any other attribute. During analysis the author realized that permission to include the business name in the map was not asked in the survey. There is some evidence that a few businesses were participating with an expectation of anonymity. As a result, all publicly available data omits the business name. The circles are individual businesses and the lines represent paths that currency takes, as expenses of the businesses. The circle in the upper left corner represents the dollar entering the economy from outside the community, and the circle in the lower right corner represents the dollar exiting the community economy.

The size of the circles was determined by the business' betweenness centrality attribute (Knocke & Yang 2008). Larger circles mean more currency flows through that business within the community. The smaller of the two green dots represents the Dunster residents, as an expense in the form of salary or other payment for services rendered. As documented in Chapter 4, no personal expenses were solicited, so 100% of the expenses of the Dunster residents are assumed to leave the community. This will be discussed further later in the chapter, under potential modeling. Additionally, any identified business that did not participate in the survey were also assumed to have 100% of their expenses leave the local community economy. This approach was deemed preferable to making inaccurate estimates.

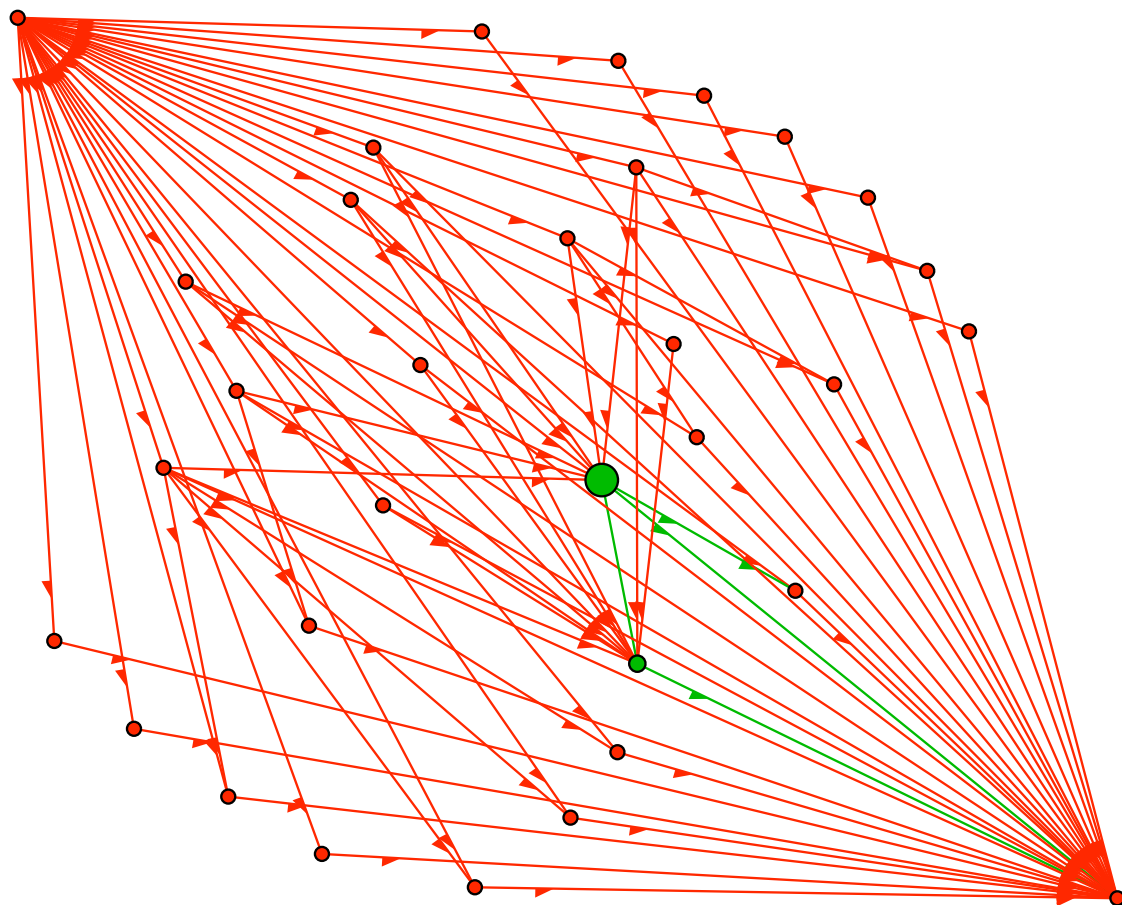


Figure 6.1.1: Map of Dunster economy

6.2 Social network analysis: Qualitative analysis

A visual inspection shows the economy of Dunster to largely be of direct flow, with no identifiable loops. Money appears to pass through Dunster, with a small amount of local expenditures. This is consistent with the survey results, in which businesses identified external expenses such as insurance, bank loans, taxes, and products for resale as their largest percentages of expenses. Quite simply, there is very little capacity in Dunster to provide services or products to other businesses in Dunster. There are at least two businesses in Dunster that do provide services and products to Dunster businesses, but they did not participate in the survey. The balance of businesses provide and obtain services and products outside of the Dunster area.

There is an additional component to capacity, and that is that many businesses in Dunster had difficulty participating in the survey because they still used paper and pencil for book keeping. This "old school" accounting method tracked expenses by category ("gas," e.g.), not vendor. Converting from category to vendor for a report with computer-based accounting is not a difficult nor a time-consuming task. This can be done by sorting on vendor instead of the default "category."

With the lack of business capacity, it is difficult to see Dunster CF obtaining secondary economic benefits from their community forest operation. This has been a challenging question posed by at least one critique of community-based natural resource management (Bradshaw 2003). While Dunster CF will be able to fund several community group efforts, and there may be some indirect benefits to local businesses through this, it seems unlikely that Dunster businesses will be able to capture direct economic benefits from the community forest. Conceivably, the Dunster CF will be able to provide reliable fiber to small mills in the Robson Valley. While this will benefit the expanded local economy, there were no mechanisms identified in which money would flow back into Dunster from this arrangement.

Figure 6.2.1 graphically represents the flow of currency into the community of Dunster and surrounding areas. Currency flows into three areas, the Robson Valley north and south of Dunster, and Dunster itself. Of the currency that flows into Dunster, it leaves either to the north or the south, into the Robson Valley or parts beyond. Of the currency that flows into the north and south parts of the Robson Valley, there are some communities in which it may recirculate, and a small amount may flow into Dunster. In general, though, the currency that flows into the Robson Valley flows back out.

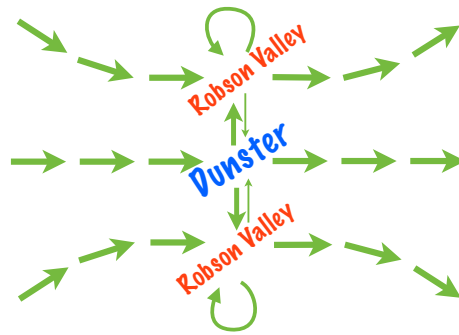


Figure 6.2.1: Currency flow in Dunster

6.3 Average number of transactions (Markov chains): Methodology

The economy outside of the community was modeled as a node, with currency flow into the community represented as aggregated expenses into the community. As business incomes were not solicited, several models were constructed to provide a range of possible values of each business' income as a percentage of the total expenses of the outside source of income into the community. These models randomized the distribution of incomes to the businesses from outside the local community. This distribution used an algorithm from Weisstein (n.d.) (see Eqn. 6.3.1) to generate a Gaussian (normal) distribution (see Figure 6.3.1a).

$$(6.3.1) \quad P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The Gaussian distribution is generated from a probability density function, where the probability of the value "x" is calculated based on the mean (μ), the value of x (which ranges from 0 to 1 with intervals equal to $1/30$ for this method, there being 30 business in the model), and σ , which is proportional to the spread of the distribution. Larger σ represent a wider, more even distribution of data.

For the modeling, σ values ranging from 0.0205 to 0.035 correspond to a peak value range of 360 to 36 times the least value, respectively. The sum of the values were normalized to 1, so with a σ value of 0.0205 the peak value was 0.072, or 7.2% of the total income into the community, while the least value was 0.0002, or 0.02%. This 360:1 ratio represents a distribution of incomes from \$1500 to \$540,000, a range the author believes is realistic based on conversations with business owners. However, this may also represent a more narrow concentration of income than may exist in the community. At the lower σ range, the 36:1 ratio represents an income range of \$5000 to \$180,000, which is likely high at the low end (\$5000), and low at the high end (\$180,000). However, the distribution is probably more likely near the middle of the spread. Using the method documented in Chapter 3, the Gini coefficient for a σ of 0.035 is 0.429 and for a σ of 0.0205 the Gini coefficient is 0.561. Given that some of the participating businesses were as small as individuals selling vegetables from their gardens, these are reasonable estimates of the distributions of incomes.

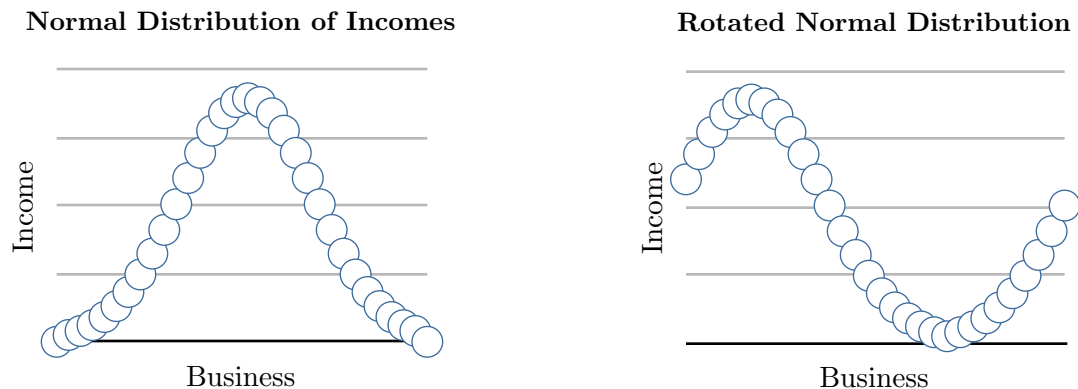


Figure 6.3.1a and 6.3.1b: Modeled distribution of incomes

This distribution of shares of currency coming into Dunster was cycled (via permutations, see Figure 6.3.1b) against different arrangements of paths through the community. Each business has an average number of transactions (path length) from that business until the dollar has exited the community. These can be arranged in different models, representing different weightings based on income. The arrangement in Figure 6.3.2a represents the paths being centered (clustered) about the longest average path length by a business in Dunster. Figure 6.3.2b represents a random arrangement, and Figure 6.3.2c represents an organization to minimize the cumulative deviation away from the average number of transactions using a simple mean. This

approach reduces instantaneous bias from having a sub-group of businesses with long or short paths.

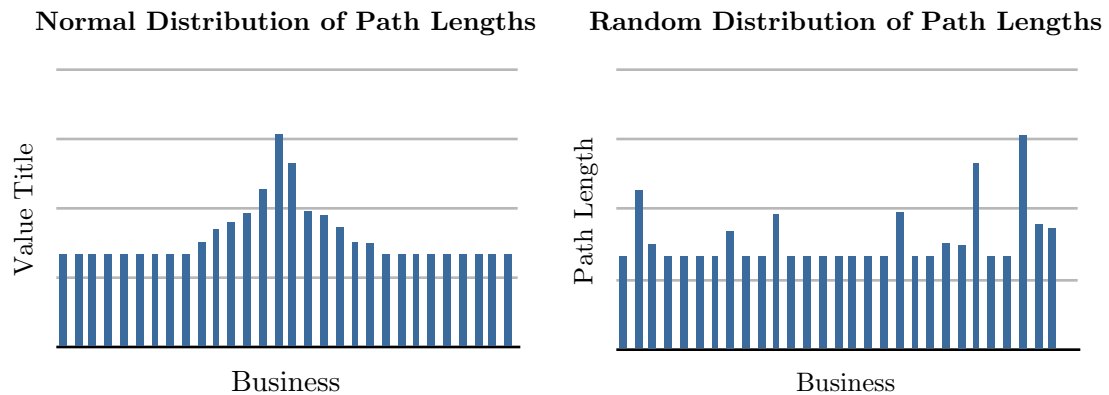


Figure 6.3.2a.

Figure 6.3.2.b.

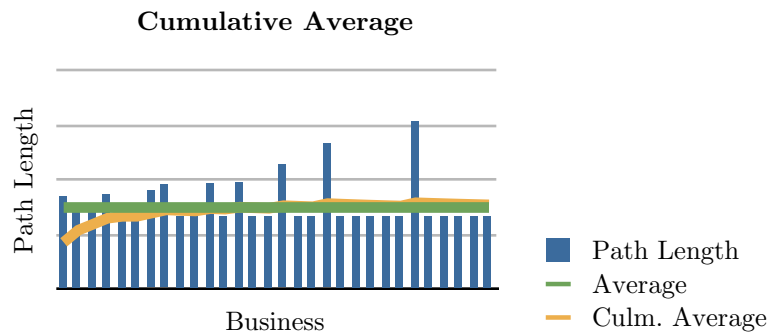


Figure 6.3.2c.

These steps were necessary to attempt to give a realistic answer and avoid bias. For example, if the businesses with the longest average number of transactions are also given the largest percentage of income into the community, the average path length for the community will be biased towards a higher number than is realistic. Conversations with the business owners in Dunster suggested that there was a significant variation of size of the business and that business' focus on spending locally. Some of the largest businesses spent significantly locally, but some also had a high level of expenses with non-local businesses. Conversely, some of the smallest businesses spent a lot locally, while others did not. Therefore, the most realistic calculation of the average number of transactions is likely to come from the range given by the distribution in 6.3.2c, which is generated through avoiding clustering of long path businesses.

6.4 Markov chains: Quantitative analysis

The data for the Dunster area economy were analyzed using custom software code to determine the average number of times a dollar circulated within Dunster before leaving the community economy. This custom software crafted the data into a Markov chain-style matrix and used

Gauss-Jordan elimination to determine the average number of transactions that occurred from the time a dollar entered the community until it left the community, as well as the number of transactions for a dollar departing from each business before it left the community. Different models were used to examine a range of scenarios. Ultimately, the economy of Dunster shows little ability to capture dollars as they pass through, and the models show that on average, the number of transactions from outside the community into the community and then back outside again is highly likely to be between 2.06 to 2.29 transactions. The author is confident the number is very near 2.2 transactions. The minimum possible number of transactions is 2.0, as one transaction is the currency being received by a Dunster business, and then one more by that business spending it outside the community.

With some efforts, this could be improved. As noted earlier, many businesses did not respond, and personal expenses were not included. If Dunster residents were to spend 20% of their expenses among other Dunster residents, 15% at the Dunster General Store, and 10% of their expenses at the Dunster CF, and the Dunster Community Forest Society were to distribute 66% of its expenses locally, while non-surveyed businesses were to distribute 30% of their expenses to Dunster residents, the average number of transactions would range from 2.45 to 2.7, using a 360:1 ratio of highest income to lowest income. However, these numbers are difficult to achieve. Dunster residents pay mortgages, insurance and power bills, and electronics much like businesses do, none of which are produced locally to Dunster. Increasing the local expenses and distribution of benefits of the Dunster CF is possible, though, and should be an area of focus for the community forest.

Model	$\sigma = 0.0205$	$\sigma = 0.035$
Measured	2.06 - 2.29	2.09 - 2.27
Enhanced	2.45 - 2.70	2.49 - 2.68

Table 6.4.1: Average number of transactions in Dunster

Table 6.4.1 lists the ranges of average number of transactions from the uniform distribution represented in Figure 6.3.2c. Repeated runs of normally distributing and randomly organizing the number of transactions for each business tended to reveal ranges equal to the least possible (2), to the highest possible, which is equal to the greatest number of transactions by a business in Dunster before a dollar spent by that business exits the community economy (2.30, see Table 6.4.2). These outcomes were not realistic, and suffered from bias.

Business #	Transactions	Business #	Transactions	Business #	Transactions
1	1	11	1	21	1.13474711027
2	1	12	1	22	1.27256175973
3	1	13	1	23	1.29791314157
4	1	14	1	24	1.34677068495
5	1	15	1	25	1.43051889453
6	1	16	1	26	1.44971514097
7	1	17	1	27	1.46783079526
8	1	18	1	28	1.70499717992
9	1	19	1.12081218274	29	1.99272740155
10	1	20	1.12992123539	30	2.29791314157

Table 6.4.2. Number of transactions before dollar spent exits community

Most of the identified businesses in Dunster did not participate in the survey. As per the established rules, these businesses were assumed to have 100% of their expenses exit the community on the next transaction (shown by a 1 in Table 6.4.2). Only one business that participated had 100% of its expenses leave the community. Of the remaining businesses, most of their expenses did leave the community on the next transaction, but some percentage did get paid out to their owners (as Dunster residents) or to other Dunster businesses. Two businesses had all of their dollars spent an additional time within the community, largely through the salaries paid to Dunster residents.

The impact of non-response survey bias is likely to be minimal. Due to the small scale of the economy in Dunster, additional responses from businesses are not likely to reveal any loops, nor are they likely to improve the overall numbers. Additional responses would likely reinforce the low number of transactions.

An unknown factor is the so-called "underground economy," where exchanges are through barter or other "off the book" transactions. Potentially these may contain some loops, but overall the impact of the underground economy in Dunster is unlikely to be high, simply due to the nature of the transactions. Typically, according to locals, underground transactions tended to be of an exchange nature, such as exchanging a chicken for some vegetables.

The results were presented to the Dunster community at-large on Oct. 14, 2012, at the first "Carbon, Climate Change, and Community Forests" conference, held in the Dunster Fine Arts School and Conference Centre. Over the two day conference, Dunster community members were

able to attend 14 presentations by 13 unique presenters, ranging from graduate students and university professors from the University of Northern British Columbia and Simon Fraser University, to representatives of the local community forests and researchers from the Ministry of Forests, Lands and Natural Resource Operations.

One consistent theme from discussions with community members about the results was their surprise that the number of transactions was so low. This should be contrasted against the advice from the community member discussed in Chapter 4 that encouraged obtaining the less exacting numbers from community members ("e.g., do you employ locals or buy product locally, and if so, how much roughly do you spend on them or how much product do you buy from locals etc etc") (Appendix D). Using subjective answers would have undoubtedly led to a higher answer than actuality. Possibly this approach would have encompassed the underground economy, but the uncertainty of the answers leaves this approach highly suspect.

6.5 Chapter conclusion

Repeated modeling attempts to increase the average number of transactions failed to reveal anything meaningful, and efforts to do so were terminated once it became obvious that unrealistic efforts would be needed to achieve anything approaching three total transactions for the community as a whole. As it stands, the Dunster economy multiplies incoming currency by about 20% after it is in the hands of a Dunster business (1.2), and with significant effort a multiplier of 30-40% is possible (1.3-1.4). The lack of business capacity, both physically and financially, is a severe impediment to growing this number.

Efforts to increase local circulation of currency should focus on identifying capacity gaps and addressing these gaps. Businesses willing to devote resources towards filling those gaps face risks in such a small economy. The Dunster Community Forest Society can assist in this process by ensuring that local value-added businesses have access to the timber within the community forest tenure area.

As an example of the challenges Dunster faces, during the above-mentioned conference, the attendees bought a concession lunch for \$10, and some made additional donations, all payable to the Dunster Fine Arts School Society. This raised approximately \$750 for the school. At the same time, several presenters and guests stayed in motels in a nearby community, for approximately \$900 total, due to the lack of lodging in Dunster.

7. Opportunities and Challenges

7.1 Adopting the metrics

The proposed metrics of economic mapping and Gini Coefficient can quantify the distribution of benefits in natural resource-dependent communities. As will be discussed in Chapter 8, the approach could be further simplified to use spreadsheets instead of a database, so the particulars of the metrics are not an impediment. This offers community forests an opportunity to quantify the distribution of their benefits, including logging access and grants to community groups. Doing so will require a local examination of possible obstacles.

7.2 How community forests in B.C. obtain revenue

There are three ways for a community forest in B.C. to obtain revenue, all of which involve logging. While community forests in B.C. do have rights to non-timber forest resources/non-timber products (NTFP) (Mulkey & Gunter 2004), the fact that the tenure areas are Crown land limits the ability to exclude the public from accessing the land (Pinkerton et al. 2008). As a result, efforts to charge admission or other tolls are not legal. Marketing NTFPs such as berries and mushrooms has not been fully established in B.C. (Davis 2011). With logging, the community forests may obtain revenue through 1) leasing cutblocks (Cathro 2004) or 2) hiring loggers to log (Cathro 2004). Hypothetically, a community could let loggers choose the areas to log and charge them an effective "stumpage" rate over and above the provincial rate. This third approach differs from the first approach in that the first approach uses a fixed price to lease an area regardless of timber harvested, while the third approach charges for the timber harvested and scaled, similar to how the Provincial government charges community forests. The Dunster Community Forest has chosen the first method (A. McLean, pers. comm., 2011).

Timber buyers pay for the timber "as delivered," which means the seller pays transportation costs unless other arrangements have been made. Consequently the distance to the buyer is a factor in the transaction. Theoretically this should favor local mills that are closer to the seller.

7.3 Community Forests face social dilemma

In attempting to achieve an optimal balance of currency circulation and distribution of benefits, a community-based natural resource management institution faces a social dilemma (Andersson & Ostrom 2008). The institution can attempt to maximize the collective outcome of the community, or it can attempt to obtain the maximum revenue possible for the sale of the resource (Cathro 2004). If the institution attempts to maximize the collective outcome of the community, inevitably a situation will arise where the institution must accept a lower than market price for the resource.

An example is easily envisioned. Two local enterprises wish to purchase timber from a community forest. One enterprise mills and works with additional local value added enterprises, while the other enterprise acts as a reseller for the logs to a large mill located outside of the community (no opportunities for local processing). The former chain offers more employment and a longer currency chain, but also has to share costs in order to achieve this longer chain. In contrast, the reseller has less overhead and can pay more for the timber while still being profitable, at the expense of possible local value-added efforts. The hypothetical community forest has a "grants" program in which community groups can apply for funding. The more the community forest receives in revenue, the more community groups that can get funded or receive larger grants. Absent quantifiable transaction costs to the contrary, the community forest institution would be behaving "rationally" in accepting the higher revenue offered by the enterprise with the shorter currency circulation chain.

This conflict was identified early in the development of the Community Forest program. In the *Community Forest Guidebook: Tools and Techniques for Communities in British Columbia*, Cathro writes

Most community forests are "market loggers," which means that they only generate revenue from the sale of logs and not from processing them. This is typical of some other smaller provincial licences, such as woodlot licences and independent logging contractors who work for BC Timber Sales.

This situation underscores the importance of maximizing the revenue from log sales (see adjacent sidebar for an example of log sale prices and harvesting costs). It also highlights the tensions between keeping the logs local (to employ local mill workers) and getting the best price for logs (by selling them outside of the community for more money). (Cathro 2004:61)

7.4 Equitable distribution of benefits in community forests

The largest economic benefit of a community forest is the access to the timber, whether through logging contracts or leasing cutblocks. An additional economic benefit, albeit much smaller, is the distribution of grants to community groups. For community forests legally structured as corporations, there may be dividends paid to the shareholders.

As can be seen in the map of the economy of Dunster in Chapter 4, the technique described in this paper shows both where benefits flow and where they do not. As discussed in Chapter 1, inequitable distribution of benefits is a significant threat to sustainable community-based management of a natural resource (Andersson & Agrawal 2011). Along with the mapping, applying the Gini coefficient to the distribution of access to the timber, as well as to logging contracts, would show a lack of equitable distribution within a community forest, if it existed. This has political implications, and potentially threatens any local elite that may have gained control of the institution managing the resource and are benefiting from the arrangement (Bardhan 2002).

Critical analysis of community-based management regularly identifies inequities in benefits and power imbalances as differences between hypothesized decentralization program and actual implementation. Leach, Mearns and Scoones (1999) suggest that community members' perception of the "collective good" is based on the members' social position. Gibson and Lehoucq (2003) correlate sustainable local management with political expediency for local politicians. Others note that democratic management locally does not spontaneously appear upon the decentralization of resource management (Platteau 2004; Bradshaw 2003), leading some researchers to examine the impact of "local tyrannies" (see Andersson and Ostrom 2008 for an overview).

If the priorities of the powerful in the community do not include a genuine desire to sustain the local resource base, then we should not expect the outcomes to differ from those of centralized management (Bradshaw 2003:5).

7.5 Incentives for improving local distribution of benefits

The proposed metrics can provide quantification of the transaction costs of not supplying locally, providing incentive and justification for community forests to reject higher bids for their timber from non-local sources, if the community forest wishes to consider a wider range of measures than price alone. This can be achieved through mapping the community economy, calculating the average number of transactions, and modeling how this would be affected by accepting different bids. If the aggregate community economy is improved by the higher, non-local bid, after taking into account the multiplier effect, then the community forest is acting rationally by accepting the higher, non-local bid. Conversely, if the aggregate community economy is better by ensuring timber circulates locally, the community forest can use the metrics to justify this decision.

Furthermore, the community forest can create incentives for local enterprises to work together in creating longer enterprise chains. If the community forest publicly states that pricing will be linked to the collective outcome for the community economy, local enterprises have assurances that they will have access to the timber at prices that allow them to partner with other local enterprises. As noted in Section 2.4, it is possible to calculate the equivalence of pricing at different recirculation rates that lead to the same aggregate economic activity for the community.

As noted in Section 6.4, small rural economies have capacity issues. As a result, the average number of transactions for each individual business may be quite low, possibly as low as 1.0, meaning that no other local businesses are sourced for goods or services during the operation of that business. If that individual business spends 20% of its expenses obtaining the timber from the community forest, that average number of transactions increases to 1.2 or more, depending on the average number of transactions for the community forest itself (the contribution to the average number of transactions of the individual business is 20% of the community forest's average number of transactions). As such, the baseline should include the expenses that go to the

community forest and changes should be measured against this baseline.

A business that is able to propose a business model that increases their average number of transactions, from 1.2 to 1.32, e.g., increases the local aggregate economic impact by \$12 for each \$100 of expenses. This can be measured against any reduction in economic impact the community forest might experience in lowering the price for the timber, in order to accommodate the business proposing such an increase in their business chain. If the community forest has an average number of transactions of 1.5, the community forest could reduce the price by 8% (\$8 per \$100) and still have the same aggregate economic impact. Conversely, if business has a longer local chain than the community forest, the benefits for reductions might be even greater than the benefits distributed by the community forests.

If the individual business spends an additional 20% of its expenses with another local business, the average number of transactions does not increase. The width of the distribution of expenses increases, though, and this can be measured with the Gini coefficient. The Gini coefficient can be calculated using the percent of expenses going to each local business and then leaving the community. For example, if 100% of the business' expenses are non-local and there are 30 businesses in the community, the GC would be 1.0. After providing the local community forest with 20% of the business' expense, the GC drops to 0.987. With an additional 20% to a local business, the GC is 0.960. These values can be calculated using the PHP script in Figure 3.2.3.

While incrementally these changes seem small, they are quantifiable differences. Therefore, given two individual businesses with identical average number of transactions (meaning their aggregate community-level economic impact is identical), the business with the lower Gini coefficient is distributing their expenses more widely. Consistent with Ostrom's principle of proportional benefits and efforts (Ostrom 1992), the community forest can justifiably offer a lower price to the business that is distributing its expenses more widely than other businesses.

The picture becomes more complex if two businesses have different average number of transactions and the business with the longer local chain distributes its expenses less widely than the business with the shorter local chain. For example, assume Business A has an average number of transactions of 1.3 and a Gini coefficient of 0.90, while Business B has an average number of transactions of 1.25 and a Gini coefficient of 0.95. The aggregate economic impact is greater for Business A, but the impact is distributed more with Business B. Conceivably the two values of average number of transactions and Gini coefficient can be viewed as dimensions of a rectangle, with the average number of transactions on one axis and the Gini coefficient on another. The area of each business' rectangle can be calculated by multiplying the two values together. As such, Business A has a combined value (area) of 1.17, while Business B has a combined value (area) of 1.18. Figure 7.5.1 depicts the differences, and as calculated above, Business B has a slightly larger area. This is by no means the only method for deciding how to handle this situation, but one that readily suggests itself.

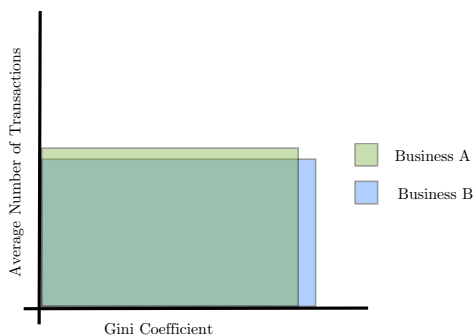


Figure 7.5.1 Comparing two businesses

7.6 Other benefits

As mentioned earlier, other economic benefits include the distribution of grants to local community groups. The metrics discussed in this paper could be used as one measure among many towards evaluating the community group proposals. Community groups that are able to engage local entities in financial transactions could be rated as more beneficial to the community on economic criteria. Obviously these are not the only criteria for which grant proposals should be measured, but there are some reasonable inferences to be made regarding the at-large community buy-in for grant proposals that omit local businesses. The same criteria for quantifying businesses can be applied to grant proposals.

7.7 Community challenges

As identified by the Dunster community members in Appendix D, and confirmed by the author's experience, many local Dunster businesses lack the ability to track expenses by vendor. The default accounting practice is to track expenses by category, such as "gas." While there are a limited number of gas stations in the Dunster area (one in Dunster, and four others within an hour's drive), the lack of an ability to discern precisely how much goes to each one of the five gas stations is a challenge to adopting the distribution metrics. For the purposes of accommodating participation, one of the businesses was allowed to estimate their division of expenses (the "business" was an individual had rather limited total expenses). On a larger scale this would create inaccuracies in the mapping and circulation metric.

For the businesses that use accounting software, sorting by vendor is not a significant task, if the user has the acumen to do so. Although some individuals in Dunster did possess this skill level, it does remain a challenge to adoption of the distribution metrics. For businesses using pencil and paper ledgers, calculating on a per vendor basis is possible but very time consuming. The author did participate in this process with two businesses. The degree of success is highly dependent on the uniqueness of the expense.

If the local community forest establishes that these metrics will be utilized in the evaluation of extraction opportunities and grant proposals, over time community members will adapt to these requirements. This may include becoming more familiar with the options available in accounting software. The community forest may need to offer training opportunities to community members. Undoubtedly, the community forest itself will need to take a leadership role in promoting the metrics as well as increasing greater focus on distributing benefits locally by all community members, not just the community forest.

7.8 Chapter Summary

The combination of economic mapping, circulation calculation, and Gini Coefficient have been shown to effectively and objectively reveal the nature of the distribution of benefits in a small resource-dependent community. These tools can be used to quantify the transaction costs of shipping raw logs outside of the local community instead of processing locally, while also establishing guidelines for pricing based on the local benefit. Community forests that wish to adopt these metrics face some capacity issues, including the inability to process logs locally, and will need to take a leadership role in addressing those capacity issues.

8. Multipliers and concluding thoughts

8.1 Benefits of natural resource extraction

The basis for modern economies involves the extraction of natural resources, whether they are inorganic materials that later provide us with the steel, gold and rare earth magnets necessary for electronics, or they are organic materials that provide us with commodity lumber for housing. The supply chain from initial extraction to the climatic product is long and has many opportunities for capturing both economic and social benefits. In a modern economy, though, the greatest profits occur at the final stages.

As discussed in the introductory chapter, the community forests in British Columbia were awarded "area-based" tenures, in which the community forest was given a specific geographic boundary; however, many of these tenure areas had been previously harvested by the major forest tenure holders (Vernon 2007), resulting in timber profiles that were often financially challenging (Pinkerton et al. 2008). Community forests pay a "land rent" for their tenure area, as well as a "royalty" for the price of the tree on the stump (known as "stumpage"). Recognizing the potential for reduced Crown royalties due to high logging costs in degraded areas (or from a lack of any logging at all), the Province of British Columbia implemented a "tabular" rate as a reduction relative to the provincial stumpage rate on January 1, 2006 (Coleman 2005a, 2005b). Typically set at 10% or 15% of the normal stumpage rate paid by major tenure holders, this difference in royalty rates has allowed some community forests to operate profitably. These reductions may violate Faustmann-Hartman rules for economically-efficient harvesting decisions (Hartmann 1976; Chang 1984; Kant 2003) by stimulating harvesting where otherwise none would occur; however, discussion of the stumpage rate is beyond the scope of this paper and is introduced only to establish the price of timber for community forests.

Consider a spruce tree of one cubic meter (m^3) in the Province of British Columbia (about the size of a telephone pole including what is buried, Pederson 2003). The province charges a community forest in the "northeast" zone of the Interior \$1.05 for that tree, or 15% of what a major tenure holder in the same area would pay (Thompson 2012). Assuming .66 net/gross ratio, where gross is the pre-finishing dimension, and 55% cubic recovery ratio (CRR), the percent of the tree that can provide lumber (Briggs 1994:19), one cubic meter of spruce contains approximately 43 2x4s of 8' foot length, suitable for framing (#2 SPF), which sell for \$2.45 each at the Home Depot in White Rock, B.C (as verified by this author on July 15, 2012). The initial \$1.05 paid to the Province by a community forest results in \$105 in product sales at Home Depot. Additional value-added occurs in the future as that 2x4 becomes an interior wall stud, which becomes a house which may sell for \$1 million in White Rock proper. If that house is 5,000 square feet, approximately 256 m^3 of timber will be necessary (assuming 55% CRR and 56.4 m^3 gross per 2,000 sq. ft., Idaho Forests Products Commission n.d.). This amount of timber can be provided with the harvesting of two hectares of land in the Robson Valley (BC MoFR 2005). Assuming \$1.05 per m^3 for 256 m^3 , through the sale of the trees, the Province, as representatives

of the entire population of British Columbia, captures approximately \$268, or 0.026% of the final value of the product, a finished house sold by a developer for \$1 million. While timber prices have fluctuated greatly in the last five years, a price of \$45/m³ for spruce delivered to a mill is within the ranges seen recently (A. McLean, pers. comm., 2011). Delivering 256 m³ to a mill results in \$11,520 to either the logger who harvested it, or to the community forest if they are selling logs directly to the mill and hiring loggers, or approximately 1.15% of the final value of the house. If all of the wood is made into 2x4s (the lowest value of framing lumber), Home Depot generates \$27,000 in sales (2x6 and 2x8 lumber is disproportionately higher in price, so this is a bare minimum based on 43 2x4s per m³ and 256 m³ for a house). As conjectured earlier, the house that is made from the lumber may sell for \$1 million. Clearly the greatest profits for natural resource extraction occur at later stages of processing.

Seller	Gross sales
Province of British Columbia	\$268 (if sold to a community forest)
Community Forest	\$11,520
Home Depot	\$27,000 (assuming only 2x4s)
Developer	\$1,000,000

Table 8.1.1: Multiplying gross sales

8.2 Multiplier effect

As discussed above, the expenditure of a dollar initiates a chain of transactions, each of which has additional economic impacts. Although each transaction involves a decreasing percentage of the original dollar, these subsequent transactions add a small but significant component to the economic impact of the original dollar. This cascade effect is known as the "multiplier effect" (Miller & Blair 2009:244). Contained within the calculation of the multiplier effect are the effects of linkage and leakage. Multipliers can differ between sectors, as labor-intensive sectors such as service industries are more likely to multiply employment in comparison to financial industries, whereas financial industries are more likely to multiply electronics sales than service industries.

In framing the question about the distribution of benefits in community-based management of natural resources, the concept of localization was discussed, along with the claimed impact of spending locally. While some statements made to the media promote a belief that "When you spend dollars locally, they are spent again in the local economy, another five to seven times" (Hudson Register Star 2010), the survey results for Dunster, B.C. businesses do not support such a statement, nor do the economic realities of free trade. As shown by the modeling exercise for Dunster, adding personal expenditures is very unlikely to bridge this difference in the average

number of transactions. This same constraint is likely to hold true for larger communities, as very few communities manufacture products consumed by local residents. Key to understanding how such a statement could be made is the concept of the multiplier effect, of which there is more than one. Unfortunately, while the multiplier effect is known to increase value in community-based management, as has been discussed in Chapter 1, there is very little literature specific to community-based management of natural resources and the multiplier effect. However, in tourism, including community-based tourism, there is significant examination of the multiplier effect.

One type of multiplier is the Keynesian multiplier, which is a measure of how much the economy increases as a result of a unit of expenditure into the economy. Archer (1984) defines this as the rate of increase of incomes (direct, indirect and induced) per unit of expenditure:

$$(8.2.1) \quad M_{\text{Keynesian}} = \frac{\text{direct} + \text{indirect} + \text{induced incomes}}{\text{unit of tourism expenditure}}$$

As noted above, the income is net, not gross. Lejárraga and Walkenhorst (2010) define the Keynesian multiplier slightly differently, focusing on the contributions to the economy instead of the income (and include induced impacts in the general economy):

$$(8.2.2) \quad M_{\text{Keynesian}} = \frac{\text{contribution to tourism economy} + \text{contribution to general economy}}{\text{unit of tourism expenditure}}$$

Through inspection, the Keynesian multipliers as defined by Archer (1984) are less than one, but as defined by Lejárraga and Walkenhorst (2006), the Keynesian multiplier may be greater than one. The other type of multiplier is the ratio multiplier, which is always greater than one. Archer (1984) identifies two different ratio multipliers: one which includes induced income (Type II), in comparison to the direct income generated by the unit of tourism expenditure, while the other does not (Type I):

$$(8.2.3) \quad \text{Type I:} \quad M_{\text{ratio}} = \frac{\text{direct} + \text{indirect incomes}}{\text{direct income}}$$

$$(8.2.4) \quad \text{Type II:} \quad M_{\text{ratio}} = \frac{\text{direct} + \text{indirect} + \text{induced incomes}}{\text{direct income}}$$

However, Lejárraga and Walkenhorst (2010) define the ratio multiplier differently:

$$(8.2.5) \quad M_{\text{ratio}} = \frac{\text{indirect Keynesian multiplier}}{\text{direct Keynesian multiplier}}$$

where

$$(8.2.6) \quad \text{direct Keynesian multiplier} = \frac{\text{contribution to tourism economy}}{\text{unit of tourism expenditure}}$$

and

$$(8.2.7) \quad \text{indirect Keynesian multiplier} = \frac{\text{contribution to general economy}}{\text{unit of tourism expenditure}}$$

This can be reduced to

$$(8.2.8) \quad M_{\text{ratio}} = \frac{\text{contribution to general economy}}{\text{contribution to tourism economy}}$$

Lejárraga and Walkenhorst (2010:418-19) state "(t)he direct Keynesian multiplier measures the first-round income generated in the tourism economy per unit of tourist spending" and "(t)he indirect Keynesian multiplier measures how much income is generated in the general economy per unit of tourist spending." Lejárraga and Walkenhorst (2010) explain the ratio multiplier as measuring how much of the tourism impact is distributed to the general economy.

8.3 Discussion about different multipliers

Clearly there are differences between the two types of multipliers, but understanding each type is compounded by Lejárraga and Walkenhorst's (2010) inconsistency in the use of terms. There is a difference between "income" and "contribution to the economy." The former is a net value (expenses are subtracted), while the latter is a gross value (total amounts, i.e., Gross National Product). However, Lejárraga and Walkenhorst (2010) use them interchangeably. Further compounding this, the Keynesian multiplier defined by Lejárraga and Walkenhorst (2010) may be greater than one (1), if the contributions to the general economy, which include indirect and induced impacts, are greater than the contribution to the tourism economy. This raises the question of whether they are referring to "income" or "sales" multipliers.

Lejárraga and Walkenhorst (2010) state in their abstract that "(t)his article presents a unifying approach for calculating indicators of tourism linkages and leakages,..." and they do offer interesting metrics for quantifying these values. However, in a prescient observation by Archer from almost 25 years earlier:

The surge of theoretical and practical research on multipliers in the 1970s has obscured the fact that, unfortunately, there are two distinct and conflicting methods of defining and expressing income multipliers. In consequence, the income multiplier values produced by recent research fall into two different and not easily compatible categories. (Archer 1984:517)

8.4 Input-output models

While examination of the benefits of tourism often only looks at primary and secondary inputs, local, regional and national economies are usually inclusive of many different industries. To examine and predict the impact of an additional dollar spent in any industry, analysts often turn to "input-output models," which were discussed in more detail in Chapter 2. As discussed, input-output models also have a "multiplier," which is calculated through algebraic manipulation of the matrix representing the sectors under consideration. This method is computationally

more intensive, but does not have the ambiguity of dual, competing versions. The multiplier is constructed from summing the outputs of upstream industries in response to a change in the demand in an industry (Miller & Blair 2009:21, 246).

$$(8.4.1) \quad l_{ij} = \frac{\text{change in output in sector } i}{\text{unit of change in final demand sector } j}$$

$$(8.4.2) \quad M_{\text{output of } j} = \sum_{i=1}^n l_{ij}$$

Input-output models usually express their values in terms of the products from the industry (kilos of aluminum per aircraft, e.g.). The focus is on the *output* of the industry, and avoids the ambiguity of differentiating between *income* and *contribution*. However, input-output models also generate income and employment multipliers using the above approach.

8.5 Using multipliers in economic impact studies

Sales multipliers use the gross value of the transaction. It may well be true that a visitor to a Grey Cup spends \$300 in merchandise and souvenirs, but this is not direct income to the retailer. The cost of purchasing the product to be sold must be deducted for this to be true reflection of the economic impact. If the manufacturer of the merchandise is in fact local, then the transaction incurs indirect impacts, which can and should be included, as discussed above.

Delpy and Li (1998), in their aptly named article "The art and science of conducting economic impact studies," identified an economic impact study of a sports event that excluded indirect impacts because the analysts felt the local linkage was low and a multiplier of 0.75 would be more appropriate than the "normal range of 1.75 to 2.25 used in most sports economic impact studies (Delpy & Li 1998:243)." As discussed above, a multiplier that could be either below 1 or above 1 is likely a confusion of two different types of multipliers. Delpy and Li (1998:243) also cite Turco (1996) as reporting "sales multipliers were erroneously used as high as 5.0-7.0." To use such a multiplier, the sale of \$300 of souvenirs at a Grey Cup would generate \$1500 to \$2100 in additional sales upstream. Those numbers are staggeringly high and quite implausible, but yet were used.

Crompton (2006) discusses the motivation behind using such high multipliers. In the highly competitive process of securing funding, the developer that produces accurate but lower numbers than a competitor may lose the job and the project. Cities that reject economic impact studies showing unrealistically high impacts and decline to build newer and bigger sports facilities may face the loss of professional sports teams to other cities that embrace the same flawed studies, with the consequences evident at the next election. As Crompton (2006) notes, in some cases persuading as few as 5% of the voters may make the difference in a narrowly contested referendum. Finding 5% of the voters who cannot discern the differences in multipliers is likely an easy task.

Conversely, even among educated voters it is difficult to discern the accuracy of the economic impact studies. The differences between income, sales and employment multipliers coupled with the differences between Keynesian and ratio multipliers (types I and II) easily challenge anyone without an economics or accounting background. Individuals within communities are concerned about the income multipliers, because that is what directly affects them (Crompton 2006). Politicians are more concerned about sales multipliers, particularly if sales tax is a significant component of revenue. Given a number by the media, the ordinary individual is likely challenged to determine what constitutes their own benefit, and just as importantly, what is their cost - but those calculations may not even have been included.

The above arguments can frame the rhetoric that surrounds localization as well. Those who desire to further localization may select numbers that present the best case to the public. Doing so, they may fall victim to the same syndrome that large-scale projects do - overstated benefits and understated costs, leading to public disenchantment. For example, the Civic Economics study cited in Chapter 1 studied the impacts of locally- and nationally- owned bookstores in Austin, Texas, finding that locally-owned bookstores contributed to the local economy \$45 per \$100 spent in contrast to \$13 from chain stores such as Borders (Civic Economics 2002). The study excluded the costs of purchasing the merchandise goods as well the cost of renting the store location. These costs are likely to dwarf the actual employment income, clearly distorting the impact of locally-owned stores.

8.6 Multipliers and the average path length

In terms of the extraction of timber, the construction of one 5,000 square foot house in White Rock, if supplied entirely with timber from a community forest, would create \$6300 in direct impacts in that community and \$140 in revenue from the sale of the natural resource to the province. There are indirect impacts that benefit the community and the province, in the form of sales of gasoline, logging equipment, and possibly consulting fees to professional foresters. There are induced impacts, in the form of groceries and clothes purchased by loggers, among other expenses. When including personal expenses, the average path length of a dollar is an aggregate multiplier, encompassing sales, employment and income multipliers. If a community such as Dunster has an average path length of 2.2, with the first transaction being to a local business, the aggregate direct, indirect and induced multiplier is the remainder, or 1.2. When modeled for personal expenses, this multiplier rose to between 1.4 and 1.7.

Multipliers can occur in different mechanisms. The timber harvest and sale is an indirect impact of the construction of the house, which obviously generates revenue for the province through sales taxes that are potentially larger than the sale of the natural resource used for materials. However, the Province of British Columbia refunds to purchasers of new homes a percentage of the "embedded" sales tax of up to \$42,500 (B.C. Ministry of Finance 2012).

8.7 Conclusions regarding localization and multipliers

Although the Dunster economy is extremely small, an economy two or twenty times larger is unlikely to have an average path length proportionally larger. This is because of limitations to local manufacturing. Unless simple staple goods such as gasoline, cereal, chainsaws and other products are made locally, the purchase of such goods represents leakage from the community economy. Within the modern economy based on free trade, very few goods are produced local to any community. Absent heavily recirculating loops, additional businesses in a larger economy merely reinforce a low multiplier through their replication of the same services that consume the same goods. This is confirmed by the discussion in Delpy and Li (1998) which finds multipliers to often be between 1.75 and 2.25 in cities large enough to host sporting events.

In a study on local impacts, Applied Economics was commissioned to examine the economic impacts of the worker's compensation insurance company SCF Arizona (Applied Economics n.d.). The authors used software from IMPLAN and multiplier values provided by the state of Arizona. Excluding the medical industry, the average "output" multiplier for the state of Arizona is 1.8. The authors state that including the medical industry the multiplier jumps to 3.7, which, if true, suggests a great deal of local provision and recirculation of the money generated through medical services.

As noted above, the Civics Economics study found a 1.45 multiplier for locally owned businesses, after gross expenses were excluded. This net multiplier is not substantially higher than the gross multiplier of 1.2 for Dunster, B.C. Truly missing from the impact, though, is determining how much of the rental costs were spent locally.

This paper concludes that while localization is beneficial and increasing the number of transactions in a localized economy is desirable, the claims of spending locally means the dollar is re-spent several more times is unrealistic and even difficult to reconcile with misusing sales multipliers. More realistic are multiplier values in the 1.2 to 2.0 range, with larger economies closer to 2.0 or slightly above. The greatest impacts possibly for spending locally are in employment multipliers, since labor is something that can be produced locally. As such, community-based management of natural resources should emphasize the connection to local employment when examining how to distribute benefits. One method for accomplishing this is to attempt to provide fiber to local mills that employ local residents. If a community forest ships a raw log out of town, all subsequent multiplier opportunities are lost.

8.8 Areas of future examination

The current method for analyzing the data uses PHP and an SQL database. However, inverting matrices is possible in Microsoft Office and NeoOffice spreadsheet applications. Apple's Numbers spreadsheet software cannot invert matrices. If the spreadsheet software can invert matrices, the average path length of a dollar in a community can be calculated using that software.

While the spreadsheet software can incorporate the effects due to loops, spreadsheet software would be unable to identify specific loops. This is a potential extension to the existing PHP code. The code could keep a table of paths and identify a loop as any path that returns to an earlier node, regardless of the probabilities of arriving there.

The existing PHP code does not handle businesses reaching the limits of their capacity during the time period being analyzed. This could impact the circulation of a dollar through the need to find alternatives. Using queuing models to incorporate time to process orders would be a logical extension to this method (Bose 2002; Breuer & Baum 2002).

The use of the McLaurin expansion to solve the Leontif inverse has been interpreted by some authors to represent influences in the economy (Miller & Blair 2009:244). I represents the initial state, A represents direct influences, and the subsequent terms represent indirect influences. As shown in Chapter 2, this does lead to an ability to calculate the multiplier effect, but it does so at a much slower convergence rate (which might be more reflective of reality than the assumed infinite number of cycles from an exact solution). The expansion of the Leontief inverse into these components is known as "structural path analysis" (Defourny & Thorbecke 1984; Duchin & Levine 2010).

The structural path analysis approach has been used to study the supply chains in forestry (Smith, Fannin, & Vlosky 2009), ecological energy and material flow in ecosystems (Suh 2005; Lenzen 2007), and even the flow of currency by sector in rural economies (Roberts 2005). The matrix of each intermediate step is representative of where the flow is, and as shown in Chapter II, could be mapped out using social network analysis (which the above authors do not do, although they do graphically depict the flows). Mapping the intermediate flows of currency would be a logical extension to this paper's focus.

While this paper has focused on using a granular approach and individualizing business, aggregating into sectors could offer some insight into specific multipliers that are not seen in the Snakes and Ladders solution. For example, the average path length approach is unable to categorically extract the employment multiplier in the community economy. Starting with the granularity of individualizing businesses while noting their industry sector, constructing an input-output model is trivial. The reverse of individualizing businesses from an input-output model is not possible.

As mentioned in Chapter 2, Kichiji and Nishibe (2008) use social network analysis to examine the circulation of community currency. Mapping communities that do and do not use community currency would offer an interesting comparison of currency flows. The challenge in community currency is that all goods brought in from outside the local community have to be purchased with bank-backed currency, which would inherently increase incentives for creating linkage and avoiding leakage.

There are many opportunities for additional metrics from the system described in this paper, from all three fields that were incorporated into this system. The betweenness and closeness centrality metrics from social network analysis can identify businesses that have significant influence on the community's economy. Markov chains form the basis of queuing theory, and queuing theory contains many metrics related to performance and efficiency (Bose 2002).

In addition to the input-output model-based Keynesian multiplier discussed above, Lejarraga and Walkenhorst (2010) established a "Linkage" metric as

$$(8.8.1) \quad \text{Linkage} = 1 + M_{\text{ratio}}$$

Through some simple algebraic manipulation, their Linkage metric can be shown to be a form of the Type I ratio multiplier identified by Archer (1984):

$$(8.8.2) \quad \begin{aligned} \text{Linkage} &= 1 + M_{\text{ratio}} \\ &= 1 + \frac{\text{indirect Keynesian multiplier}}{\text{direct Keynesian multiplier}} \\ &= 1 + \frac{\frac{\text{contribution to general economy}}{\text{unit of tourism expenditure}}}{\frac{\text{contribution to tourism economy}}{\text{unit of tourism expenditure}}} \\ &= 1 + \frac{\text{contribution to general economy}}{\text{contribution to tourism economy}} \\ &= \frac{\text{contribution to tourism economy} + \text{contribution to general economy}}{\text{contribution to tourism economy}} \end{aligned}$$

Lejarraga and Walkenhorst (2010) also defined a "Leakage" metric based on the Keynesian multiplier:

$$(8.8.3) \quad \begin{aligned} \text{Leakage} &= 1 - M_{\text{Keynesian}} \\ &= 1 - \frac{\text{contribution to tourism economy} + \text{contribution to general economy}}{\text{unit of tourism expenditure}} \end{aligned}$$

Lejarraga and Walkenhorst (2010) note that the linkage and leakage metrics do not have to sum to 1, and they are not "mirror images of each other" (Lejarraga & Walkenhorst 2010:420). Connecting these two metrics to the system described in this paper would yield additional information. Of note, the linkage and leakage metrics from Lejarraga and Walkenhorst (2010) are using dollar amounts, which this system does not collect. Some adjustment to their metrics would be necessary to be fully compatible.

The proposed utilization of the Gini coefficient and average path lengths in Chapter 3 does not consider branching, which adds a "width" to the supply chain. This should be considered. Additionally, as mentioned in Chapter 3, the use of the social network analysis metric "betweenness centrality" could be incorporated into calculating the Gini coefficient for an entire economy.

8.9 Final conclusions

The issue of capturing benefits from the extraction of natural resources is an extremely important one. The value of the natural resource in raw form is a small fraction of the value in its processed and finished form. The high volume-low profit nature of raw material extraction leaves little chance to capture value, emphasizing the need to circulate the benefits as close to the extraction site as possible. As is anything involving money, this circulation is at risk for capture by local elites, and sustainable management of the natural resource inherently requires equitable distribution.

The use of Markov chains can quantify how well the benefits circulate, while the use of the Gini coefficient can quantify how well the benefits are distributed. Markov chains allow the calculation of an outcome based on probability, and are essential to modeling the probability a dollar will go from one business to another, based on the percentages of expenses for a business. The Gini coefficient is a method for quantifying the distribution of the slices of a pie among the diners. By quantifying this value, the community can set a socially acceptable level of concentration of benefits, ideally in conjunction with the idea of proportionate benefits for the amount of effort put into the resource. Those that put the most into managing the resource can be rewarded by getting the largest proportion of the benefits, within a limit that ensures the benefits are available widely. In the case of Markov chains, the mathematics is identical to social network analysis and input-output models, allowing additional tools to be brought to the analysis of the distribution of benefits. The use of structural path analysis can show how the benefits are circulating, whether directly, indirectly, or through induced impacts.

The actual implementation of these tools faces some practical challenges, however. There are on-the-ground realities to consider, including the lack of industrial and professional capacity in rural natural resource dependent communities. In Dunster both of these challenges were present, where businesses had difficulty providing the necessary information because their accounting books were still done with pencil and ledgers, and there was almost no local capacity to do value-adding to the raw timber. Their choices were largely limited to how far were they going to ship the raw logs, and buyers farther away might pay more than local buyers, enough to offset the extra delivery costs.

The tools are quantitative, and they do require a different approach to organizing accounting records. With computer-based accounting software, this can be handled in reports simply by sorting on vendor instead of category. Even the emphasis on category instead of vendor directs thought away from with whom the money is being spent with and towards how the money is spent instead. Our mindsets are not towards localization.

This is not to say these tools are not useful. Rather, the opposite is true. Even a superficial examination of studies of the economic impacts of localization reveals disconnects with public statements about how much communities benefit from that localization. Uneducated estimates

of how often a dollar circulates are radically wrong, which does not help the localization effort. Combining social network analysis and Markov chains creates the ability to identify linkage and leakage within the local community and quantitatively determine if changes are in the best direction. This can be applied to the distribution of grants by community groups managing the natural resource, or to evaluate the costs of offering tax breaks to new industry.

Sustainable management of natural resources requires the management of people. We have inherent ideas about fairness, and when that fairness is missing everyone is worse off. As discussed in Chapter 3, social ills such as teen pregnancy, alcoholism and mental illness all increase in distorted societies. It may seem antithetical to measure fairness, and this paper does not really attempt to do so. However, by measuring the circulation of a dollar in a community and measuring the distribution of benefits of extracting natural resources, this paper does attempt to create benchmarks by which communities can measure themselves.

"If you can't measure it, you can't manage it." -- Peter Drucker

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Appendix A: Finding the inverses of matrices by determinants and adjugates

While multiplication of matrices proceeds as one might expect, there is no division operation in linear algebra. Hence, while

$$(A.1) \quad \frac{x}{x} = x \cdot \frac{1}{x} = 1$$

the equivalent in linear algebra for a given matrix B (where I is the identity matrix) is not true.

$$(A.2) \quad \frac{B}{B} \neq B \cdot \frac{1}{B} \neq I$$

Rather, the identity matrix I is obtained through multiplying a matrix by its *inverse* (McMahon 2006:52).

$$(A.3) \quad B^{-1} \cdot B = B \cdot B^{-1} = I$$

The inverse of a matrix B can be calculated from

$$(A.4) \quad B^{-1} = \frac{1}{\det|B|} \text{adj}(B)$$

where $\det|B|$ is the determinant of matrix B and $\text{adj}(B)$ is the adjoint matrix of matrix B (Miller & Blair 2009:693). The adjoint matrix is also known as the adjugate matrix (McMahon 2006:72).

The determinant of a matrix is found through a lengthy process of summing the multiplication of each element of the matrix times the element's cofactor:

$$(A.5) \quad \det|B| = \sum_{i=0}^n b_{ij} c_{ij}$$

The cofactor of a matrix element is found through multiplying the minor m_{ij} of element b_{ij} by an alternating positive or negative sign determined by the location of b_{ij} in the matrix B :

$$(A.6) \quad c_{ij} \equiv (-1)^{i+j} m_{ij}$$

The minor m_{ij} is found through the determinant of the matrix formed by removing the row and column in which b_{ij} appears in the matrix B . For example, in a 3x3 matrix (labeled B),

$$(A.7) \quad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

the minor matrix M_{32} is

$$(A.8) \quad M_{32} = \begin{bmatrix} b_{11} & b_{13} \\ b_{21} & b_{23} \end{bmatrix}$$

and the determinant m_{32} is

$$(A.9) \quad m_{32} = b_{11}b_{23} - b_{13}b_{21}$$

From Eqn. (A.6),

$$\begin{aligned}
 & i + j = 5 \\
 & (-1)^5 = -1 \\
 \text{(A.10)} \quad & c_{32} = -1(b_{11}b_{23} - b_{13}b_{21})
 \end{aligned}$$

This is repeated for all elements of B to calculate the determinant.

The adjoint matrix is found through forming the matrix of cofactors calculated from equation. A.6 and taking its transpose (McMahon 2006:72).

$$\text{(A.11)} \quad C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n-1} & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n-1} & c_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ c_{n-11} & c_{n-12} & \cdots & c_{n-1n-1} & c_{n-1n} \\ c_{n1} & c_{n2} & \cdots & c_{n-1n-1} & c_{nn} \end{bmatrix}$$

$$\text{(A.12)} \quad \text{adj}(B) = C^T = \begin{bmatrix} c_{11} & c_{21} & \cdots & c_{n-11} & c_{n1} \\ c_{12} & c_{22} & \cdots & c_{n-12} & c_{n2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ c_{1n-1} & c_{2n-1} & \cdots & c_{n-1n-1} & c_{nn-1} \\ c_{1n} & c_{2n} & \cdots & c_{n-1n-1} & c_{nn} \end{bmatrix}$$

When both the determinant and adjoint matrix of matrix B have been found, the inverse is found through Eqn. A.4 above.

Appendix B: Proof of equivalence of social network analysis, Markov chain transition and input-output matrices

Following the methodology explained in Miller and Blair (2009:244), the Leontief inverse matrix is composed of

$$(B.1) \quad L = (I - A)^{-1}$$

where I is the identity matrix and A is composed of the technical coefficients a_{ij} . These technical coefficients are built from the value of the inputs divided by the total value of the outputs for the sector.

$$(B.2) \quad a_{ij} = \frac{\text{value of inputs from sector } i \text{ bought by sector } j}{\text{total value of the output of sector } j}$$

If z_{ij} is the value of the inputs from sector i bought by sector j and x_j is the total value of the output of sector j , the expression becomes

$$(B.3) \quad a_{ij} = \frac{z_{ij}}{x_j}$$

It is not a requirement that technical coefficient values be normalized, i.e.,

$$(B.4) \quad \frac{\sum_i z_{ij}}{x_j} \neq 1$$

but these values are all less than 1. In an input-output model, goods flow from i to j ; by inference, currency flows in reverse, from j to i .

Let s_{ij} be the percent of sector i 's expenses that go to each sector j . If the total expenses of sector i are included, including profit as an absorbing value on the diagonal ($i = j$), then the scale of the coefficients a and s are the same.

$$(B.5) \quad s_{ij} = \frac{\text{cost of inputs from sector } j \text{ bought by sector } i}{\text{total costs of sector } i}$$

Social network analysis creates matrices using the strength of the relationship from i to j . These strengths can be normalized.

This difference in direction (flow of goods vs. flow of currency) means that the matrix of technical coefficients A from input-output models is the transpose of the matrix mapping the strength of the relationships in social network analysis, i.e.,

$$(B.6) \quad a_{ij} = s_{ji}$$

$$(B.7) \quad A = S^T$$

The multiplier effect of an increase in demand of a sector in an input-output model is derived from the sum of the elements in the first *column* of the Leontief inverse matrix (Miller & Blair 2007:245). The average path length from the first node in a Markov chain until the exit node is

the sum of the elements in the first *row* of the Leontif inverse matrix, although the S matrix is minus the row and column associated with the exit node (Althoen, King and Schilling 1993). As such, it is necessary to determine if these are, in fact, the same values. A few rules about linear algebra are necessary:

The transpose of the identity matrix is the identity matrix:

$$(B.8) \quad I = I^T$$

The transpose of the sum (or difference) of two matrices is equal to the sum (or difference) of the transposes of the matrices (McMahon 2006:46):

$$(B.9) \quad (M + N)^T = M^T + N^T$$

The transpose of the inverse of a matrix is equal to the inverse of the transpose of the matrix (McMahon 2006:54):

$$(B.10) \quad (M^{-1})^T = (M^T)^{-1}$$

Let S represent the social network analysis matrix comprised of the strengths as currency flowing from i to j as s_{ij} and let A represent the input-output matrix comprised of the technical coefficients of goods flowing from i to j as a_{ij} . From (B.7):

$$(B.11) \quad A = S^T$$

Construction of the Leontief inverse matrix in input-output model:

$$(B.12) \quad L = (I - A)^{-1}$$

or for Markov chain-based social network analysis:

$$(B.13) \quad L' = (I - S)^{-1}$$

Let

$$(B.14) \quad D = I - S$$

Substituting into (B.13),

$$(B.15) \quad L' = D^{-1}$$

Take the transpose of both sides,

$$(B.16) \quad (L')^T = (D^{-1})^T$$

From (B.10), the transpose of the inverse of D is now the inverse of the transpose of D ,

$$(B.17) \quad (L')^T = (D^T)^{-1}$$

Reversing the substitution from (B.14)

$$(B.18) \quad D^T = (I - S)^T$$

From (B.9)

$$(B.19) \quad (I - S)^T = (I^T - S^T)$$

therefore

$$(B.20) \quad D^T = (I^T - S^T)$$

and from (B.8) and (B.11)

$$(B.21) \quad (I^T - S^T) = (I - A)$$

Substituting into (B.20)

$$(B.22) \quad D^T = (I - A)$$

Taking the inverse of both sides

$$(B.23) \quad (D^T)^{-1} = (I - A)^{-1}$$

Using (B.17)

$$(B.17) \quad (L')^T = (D^T)^{-1}$$

and (B.23)

$$(B.24) \quad (L')^T = (I - A)^{-1}$$

which concludes with using (B.13)

$$(B.25) \quad (L')^T = L$$

The Leontief inverse matrix of social network analysis is the transpose of the Leontief inverse matrix of input-output models. The *rows* of the Leontief inverse matrix of social network analysis are equal to the *columns* of the Leontief inverse matrix of input-output models, and therefore, the average path length of a social network analysis-based Markov chain is equal to the multiplier effect of a normalized input-output model.

Appendix C: Economic mapping of Dunster and surrounding areas

This project aims to map the business community of Dunster. The data will be used to map the path a dollar takes from when it arrives at a Dunster business until that dollar leaves the Dunster community. This project is being done by Tim Kelly, of Simon Fraser University, and the Dunster Community Forest Society (DCFS), with funding being provided by the Future Forest Ecosystem Science Council (FFESC). If sufficient data is provided, the results will include a comparison of the Dunster economy before and after the community forest agreement was awarded. Our goal is to have the results available to the general public no later than Oct. 31, 2011.

Your participation in this survey is very important and greatly appreciated by DCFS, Tim Kelly, and FFESC. Please complete the following survey, which will ask you, a Dunster business owner, for the percentages of your business expenses that go to businesses you identify. This survey does not ask you for dollar amounts, only percentages, and does not include personal expenses, only business expenses. Please return the survey to Tim Kelly, [Removed], [Removed], BC [Removed] or call Tim at [Removed]. If you need assistance in preparing the data, please contact Tim, who is volunteering to assist and will keep all dollar values confidential.

Please list where your business expenses go and the percentage of your expenses that go to that business. Percentages of employee salaries, including your own salary and profit you invest, go on the three lines labeled "Dunster employees," "[Removed] employees," and "Other employees." Please identify the "other" locations. Please include the percent you pay to provincial and federal revenue agencies in taxes and licenses on separate lines. Ideally your total should equal 100%, and please list any business in which more than 0.1% of your expenses went to. Please be specific about either the address, including the town, or the phone number, if you do not know the specific address. This will be used to identify "local" and "non-local" businesses.

An example (private) calculation: \$2500 in profit, \$10,000 in expenses including profit = $.25 * 100 = 25\%$

Example entries:

<u>Business</u>	<u>Address</u>	<u>Phone Number</u>	<u>2007</u>	<u>2010</u>
self	Dunster-Croyden Rd.	(250) 968-5555	26%	25%
[Removed]	[Removed]	[Removed]	22%	17%
Dunster General Store	Dunster-Croyden Rd.	(250) 968-4488	11%	19%
[Removed]	[Removed]	[Removed]	9%	8%
[Removed]	Hwy 16		14%	15%
[Removed]	[Removed]		5%	4%
Canada Revenue Agency	Kelowna, B.C.		5%	12%
B.C. Ministry of Finance	Victoria, B.C.		7%	0%

Appendix D: Correspondence with community member

July 15, 2012 initial email from community member

Hi Tim,

We met yesterday or the day before at the store when you came with the surveys. I've now had some time to mull it over and have the following thoughts, forgive me if I am blunt but you need to hear and think about these things:

I had a quick look at the survey and felt it was VERY complicated and wasn't sure how I would fill it out. Most businesses track expenses by category and not vendor so to pick out how much (for example) of the store's expenses go to locals vs farther afield is a huge job to pull out of the books. And to write down every vendor (from our main suppliers to travelling salesmen) is a bigger chore than filing tax returns and low on my priority list. I think what I will ultimately send you (if I get around to it at all) will be really lame estimates (so much to Dunster, so much to [Removed], the rest - most of it because there are so few local sources of groceries - farther afield), as even though I have the books on computer I don't know if the program will pull up how much we have spent in a given year for a certain vendor, and even that is not an expense per se as the goods go into inventory and aren't counted as expenses until they are sold. And when we do inventory every year we don't break it down into vendor or even category at all, it's enough just to count everything.

So you might think about another message via Pete's list that might help anyone who dared tackle your survey and try to make the instructions clearer (and suggest you are open to very rough estimates).

Also, apart from the store, the vast majority of "businesses" in Dunster are home based. Everything from farms (many) to French Horn makers to tree planters to myself (I edit for a living when I am not at the store, almost entirely for institutions outside of Canada). Something missing from your survey is what money comes IN to Dunster from outside of Dunster because of our various professions (myself and said French Horn makers and others I can think of included). Many people have more than one occupation to get by. I'm not sure that half of them would even consider themselves a "Dunster business" when walking into the store thus would not even pick up your survey. So that's another thing to consider in another message to the list.

And sorry but a cookie or two as incentive just doesn't cut it (didn't want to say that when you were in but I thought it) - I'm 6 months behind on the store books thus facing fines for not filing HST returns on time, so am far more likely to use what time I have catching up rather than trying to pull percentages out of the books in exchange for a cookie (just as an example).

At the same time I think what you are doing is interesting and admirable, I'm just not sure you are using the right approach. I've been there done that with the master's thesis (completely different situation and topic but you have to get it done) so I can empathize.

In closing, sorry if this message seems harsh, I think a better approach would be to 1) identify Dunster businesses (we can help you with that); 2) make appointments to talk to those people face to face or by telephone; 3) ask them a very short set of questions (e.g., do you employ locals or buy product locally, and if so, how much roughly do you spend on them or how much product do you buy from locals etc etc) and be prepared for rough estimates. I think you would get a lot farther with that than with the survey. But I may be wrong.

Good luck with it all, I don't mean to be negative but wanted you to know what you are up against here.

Cheers,
(removed)

July 16, 2012 Response by author

Hi (removed)!

Awesome feedback! Thank you very much. There is, unfortunately, a method to my madness. I do expect that many people will feel this is too much work. If you have a lot of vendors, it could be quite a task. I can simplify some by asking that business owners list Dunster businesses and their percentages, and then give me a percentage for the amount they know leaves the community (defined by the 968 prefix). The works as long

as no one (DCFS) asks for an expansion to include [Removed]. Personally I do not think much [Removed] money comes back to Dunster, but I could be wrong. I do know I will not get much response at all from [Removed] on this survey.

I weighed asking about money coming in, and unfortunately I feel that the inaccuracies in this would be too high (and there would be big gaps). I am also hoping that it might be possible to build an ongoing database of linkages, so that larger geographical regions could be included, over time. That would start to reflect how some of the money comes in. It does turn out that while diversity in income is important, it's the expenses that determine how that income is retained within the community.

It is hard to explain what I am doing in a short email (I can write really long emails so I really, really appreciate your time that you took to write me), but basically I have developed a mathematical model that can calculate the number of times the dollar circulates, if I know the paths and can construct the connections that recirculate the money. There is an adage that says money spent locally circulates seven times. Well, I've done my homework and I think this number is made up. It appears to come from input-output models, which aggregate dollars into categories, like you do, but by aggregating categories the paths are lost. If there is a non-uniform distribution of dollars into a category (like one vendor getting 90% of the input), then the multiplier effect doesn't benefit everyone equally. That one vendor might have the shortest path out of the community - for example, an increase of \$100,000 in retail has one level of impact when it goes to a local vendor and a completely different level of impact when it goes to Walmart. If all I have is category (say, retail), though, I can not differentiate how it benefits the community. If I have the paths I can show how Walmart drains the community by having a real short path out of the community. Conversely, a "sustainable community" ideally has a lot of paths that recirculate back into the community.

It appears possible to construct an input-output model from the path data, but I can not see a way to construct the path data from an input-output model. That means there's a good chance that some portions of claims of beneficial impact of government spending are probably wrong. What is different with the path model is that I can show Dunster Community Forest Society how to distribute the benefits like logging opportunities to optimize both distribution and paths at the same time (but not maximize, as one can not maximize both at the same time).

The cookie was more of an attempt at being light-hearted than being serious. It'll have to be a moral suasion effort than one cookie to truly reward the survey takers. I actually hope to speak with as many business owners that will sit with me, but I am hoping that giving them the survey well ahead of time will let them have time to calculate their numbers.

If I offer that survey takers don't have to be specific about vendors that are outside the Dunster community, but still need to give me a number for the percentage of their expenses that go outside the community, do you think that will ease the burden sufficiently?

thanks again for the awesome and honest feedback!
tim

July 21, 2012 response by community member

Good luck at the market - good that you will be there but don't expect to have a lot of time to "chat" while you are madly flipping burgers (or whatever job you are given). You may want to use what time you have to set up longer interview/discussion time with those open to it.

I've had a fair bit of (informal) feedback about your survey at the store and most of it is similar to mine (survey is confusing, businesses don't categorize by vendor but by category and most don't have the time to try to separate local vs other purchases, and nobody has the time to dig in their books for even the most rudimentary numbers/estimates) but also privacy concerns (one anyway). Just so you know.

Having gone through the process of getting a master's degree (though far afield from yours) I empathize but still think this was poorly thought out. I'd like for you to get enough valid data to make your thesis work (or maybe it will be a great thesis about how not to do it) so listen to all the emails you have received (not just mine) and what people tell you tomorrow and maybe rethink how you can make this work for you and the community.

Over and out.
(removed)

Appendix E. SQL and PHP code

Matrix.php

```
#!/usr/bin/php -q
<?php

require_once "sql.php";

$g_nodes[0] = "Source";

function IdentityMatrix($rows, $cols) {

    // construct the identity matrix
    //   $I = array();

    $I = array_fill(0, $rows, array_fill(0, $cols, 0));
    for ($i = 0; $i < $rows; $i++) {
        // $I[$i] = array();
        // for ($j = 0; $j < $cols; $j++)
            // $I[$i][$j] = 0;
        $I[$i][$i] = 1;
    }

    return $I;

}

function Minor($arry, $row_pos, $column_pos) {

    $m = array();

    $no_row = Array_Delete_Element($arry, $row_pos);
    $rows = count($no_row);
    $cols = count($no_row[0]);

    for ($i = 0; $i < $rows; $i++)
        $m[$i] = Array_Delete_Element($no_row[$i], $column_pos);

    return $m;

}
```

```

function Transpose($arry) {

    $rows = count($arry);
    if ($rows > 0)
        $cols = count($arry[0]);
    else
        return null;

    $m = array_fill(0, $cols, array_fill(0, $rows, ''));

    for ($i = 0; $i < $rows; $i++) {
        for ($j = 0; $j < $cols; $j++) {
            $m[$j][$i] = $arry[$i][$j];
        }
    }

    print_array($arry);print("\n");
    print_array($m);

    return $m;

}

function MatrixMultiply($m, $n) {

    $rows_m = count($m);
    $cols_m = count($m[0]);

    $rows_n = count($n);
    $cols_n = count($n[0]);
    $p = array_fill(0, $rows_m, array_fill(0, $cols_n, ''));

    if ($cols_m != $rows_n)
        return null;

    // the multiplication is m x n
    // final array will have $rows_m rows
    // and $cols_n columns

    for ($i=0; $i<$rows_m;$i++) {
        for ($j=0; $j<$cols_n; $j++) {

```

```

        $a = 0;
        for ($k=0; $k<$cols_m;$k++) {
            $a = $m[$i][$k] * $n[$k][$j] + $a;
        }
        $p[$i][$j] = $a;
    }
}

return $p;
}

function MatrixAdd($m, $n) {

    $rows_m = count($m);
    $cols_m = count($m[0]);

    $rows_n = count($n);
    $cols_n = count($n[0]);

    //print("\$rows_m: $rows_m, \$cols_m: $cols_m\n");
    //print("\$rows_n: $rows_n, \$cols_n: $cols_n\n");

    if (($cols_m != $cols_n) || ($rows_m != $rows_n)) {
        print("mismatch\n");
        return null;
    }

    $p = array_fill(0, $rows_m, array_fill(0, $cols_m, ''));

    for ($i=0; $i<$rows_m;$i++) {
        for ($j=0; $j<$cols_m;$j++) {
            $p[$i][$j] = $m[$i][$j]+$n[$i][$j];
        }
    }

    return $p;
}

function Get_Type($var) {

    if (is_numeric($var))

```

```

        return "numeric";
    elseif (is_string($var))
        return "string";
    elseif (is_array($var))
        return "array";
    elseif (is_bool($var))
        return "bool";
    elseif (is_null($var))
        return "null";
    else
        return "unknown";
}

function StrCompare($str1, $str2) {

    return (strcmp($str1, $str2) == 0);

}

function Find_In_Array($value, $arry, $after=0) {

    $t = Get_Type($arry);
    if (! StrCompare($t, "array"))
        return -1;

    $a = array_slice($arry, $after);
    $pos = strpos("@", $value);
    if ($pos === false) {

        if (false === in_array($value, $a))
            return -1;
        else {
            $b = array_search($value, $a);
            return $b;
        }

    } else {
        // have to use strpos
        $next = strpos("@", $value, $pos);
        if ($next === false) {
            if ($pos == 0)
                $value = substr($value, 1);
        }
    }
}

```

```

        else
            $value = substr($value, 0, $pos-1);

            while (list($key, $v) = each($a)) {
                if (strpos($value, $v) !== false)
                    return $key;
            }
        }
    }
}

```

```

function Array_Delete_Element($arry, $element, $count=1) {

    if ($element == 0)
        $b = array_slice($arry, 1);
    else {
        $c = array_slice($arry, 0, $element, true);
        $d = array_slice($arry, $element+1);
        $b = array_merge($c, $d);
    }

    return $b;
}

```

```

function print_array($arry) {

    $rows = count($arry);

    for ($i = 0; $i < $rows; $i++) {
        $cols = count($arry[$i]);
        if (($cols == null) || ($cols == 0)) {
            print("zip");
        } else {
            for ($j = 0; $j < $cols; $j++)
                print($arry[$i][$j] . " ");
            print("\n");
        }
    }
}

```

```

function GaussJordanReduction($chain) {

```

```

$rows = count($chain);
$cols = count($chain[0]);

$chain = Minor($chain, $rows-1, $cols-1);
//$chain = Transpose($chain, 0, 0);

//print("\n");print_array($chain);

$rows = count($chain);
$cols = count($chain[0]);

$I = IdentityMatrix($rows, $cols);

$M = array();
$GJ = array();
for ($i = 0; $i < $rows; $i++) {
    $M[$i] = array();
    $GJ[$i] = array();
    for ($j = 0; $j < $cols; $j++)
        $M[$i][$j] = $I[$i][$j] - $chain[$i][$j];
    $GJ[$i] = array_merge($M[$i], $I[$i]);
}

//print("\n");print_array($GJ);
//print("\n");

$rows = count($GJ);
$gj_cols = count($GJ[0]);

for ($jj = 0; $jj < $cols; $jj++) {
    // $jj tracks the current column
    if ($GJ[$jj][$jj] != 1) {
        $n = $GJ[$jj][$jj];
        for ($j = $jj; $j < $gj_cols; $j++)
            $GJ[$jj][$j] = $GJ[$jj][$j]/$n;
    }

    $pivot_row = $GJ[$jj];
    if ($pivot_row[$jj] == 0)
        print("have a zero in the pivot location for row $jj\n");
}

```

```

        for ($ii = $jj+1; $ii < $rows; $ii++) {
            $lower_row = $GJ[$ii];
            //print_array($lower_row);
            //for ($j = 0; $j < $i-1; $j++) {
            if ($lower_row[$jj] != 0) {
                $v = -($lower_row[$jj]/$pivot_row[$jj]);
                for ($j = $jj; $j < $gj_cols; $j++) {
                    $m = $v * $pivot_row[$j];
                    $lower_row[$j] = $lower_row[$j] + $m;
                    $m = 0;
                }
                $GJ[$ii] = $lower_row;
            }
        }

//
    }

//print("\$jj: $jj\n");
//print("\n");print_array($GJ);
//print("\n");
    }
}

// go back up in reverse
for ($jj = $cols-1; $jj > -1; $jj--) {
    $pivot_row = $GJ[$jj];
    for ($ii = $jj - 1; $ii > -1; $ii--) {
        $supper_row = $GJ[$ii];
        if ($supper_row[$jj] != 0) {
            $v = -($supper_row[$jj]/$pivot_row[$jj]);
            for ($j = $jj; $j < $gj_cols; $j++) {
                $m = $v * $pivot_row[$j];
                $supper_row[$j] = $supper_row[$j] + $m;
                $m = 0;
            }
            $GJ[$ii] = $supper_row;
        }
    }

//print("\$jj: $jj\n");
//print_array($GJ);
//print("\n");
    }
}
}

```



```

        //print("\n");print_array($GJ);

        return $GJ;
    }

function LocationNumber($location_name) {

    $result = pg_query($GLOBALS["db_conn"], "SELECT uid FROM geo_loc where
name='$location_name';");

    if ($result != null) {
        $row = pg_fetch_array($result);
        $loc_num = $row[0]; //print("\$loc_num: $loc_num\n");

    } else
        $loc_num = -1;

return $loc_num;
}

function GenerateSubchain($location, $model, $order) {

global $g_nodes;

    $loc_string = "{}";
    $loc_num = LocationNumber($location[0]);
    $loc_string = $loc_string . strval($loc_num);

    for ($i = 1; $i < sizeof($location); $i++) {
        $loc_num = LocationNumber($location[$i]);
        $loc_string = $loc_string . ", " . strval($loc_num);
    }

    $loc_string = $loc_string . "}'";

    // get the listings of nodes first
    $result = pg_query ($GLOBALS["db_conn"], "select * from markov_chain($loc_string,
'$model') where src='Source';");
    if ($result != null) {
        $index = 1;
        while ($row = pg_fetch_array($result)) {

```

```

        // src_uid | src | dest | str | total | pct
        $g_nodes[$index] = $row["dest"];
//print($g_nodes[$index]."\n");
        $index++;
    }

    $g_nodes[$index] = "Sink";

    $sz = sizeof($g_nodes);
    // build the $sz x $sz array with 0 as default value
    $filler = array();
    $filler = array_pad($filler, $sz, 0);
    $subchain = array();
    $subchain = array_pad($subchain, $sz, $filler);
//print("\$sz: $sz\n"); print(sizeof($subchain[0]) . "\n");
    //now build the array
    $result = pg_query($GLOBALS["db_conn"], "select * from
markov_chain($loc_string, '$model');");
    if ($result != null) {
        // first row is Source, just ignore
        //$row = $sql_fetch_array($result);

        while ($row = pg_fetch_array($result)) {
            // src_uid | src | dest | str | total | pct
            $src = $row["src"];
            $dest = $row["dest"];
            $pct = $row["pct"];

            if ($order == null) {
                // Find_In_Array destructive of arrays?
                $t_nodes = &$g_nodes;
                $src_i = Find_In_Array($src, $t_nodes);
//print("\$src: $src \$src_i: $src_i ");
                $t_nodes = &$g_nodes;
                $dest_i = Find_In_Array($dest, $t_nodes);
//print("\$dest: $dest \$dest_i: $dest_i\n");
            } else {
                $t_nodes = &$order;
                $src_i = Find_In_Array($src, $t_nodes);
//print("\$src: $src \$src_i: $src_i ");
                $t_nodes = &$order;
                $dest_i = Find_In_Array($dest, $t_nodes);

```

```

//print("\$dest: $dest \$dest_i: $dest_i\n");
    }

    $subchain[$src_i][$dest_i] = $pct;
}
$t_nodes = &$g_nodes;
$src_i = Find_In_Array("Sink", $t_nodes);
$subchain[$src_i][$src_i] = 1.0;

// remove Source distribution
$subchain = Array_Delete_Element($subchain, 0);

if (0 == 1) {
    $org_subchain[0][0] = 0.0;
    $org_sum[0] = 0.0;
    $sz = count($i_sort);
    for ($cc = 0 ; $cc < $sz; $cc++) {
        $org_subchain[$cc] = $subchain[$i_sort[$cc]];
        $org_sum[$cc] = $sm[$i_sort[$cc]];
    }
    //print($org_sum[$cc] . "\n");
    //print_r($sm);
}
} else
    return array(array("error - no nodes"));

} else
    return array(array("error - no source nodes"));

return $subchain;
}

function IterateOverSubchain($subchain_r, $perm_r, $min_sum, $max_sum) {

global $g_nodes;

// check our cumulative distribution function
$cycle_r[0][0] = 0.0;
$cycle_r[0] = $perm_r[0];
$chain_r = array_merge($cycle_r, $subchain_r);
//print_array($chain);

```

```

$rows = count($chain_r);
$cols = count($chain_r[0]);

// loop through the permutations
$columns = $cols;
//print("\$columns: $columns\n");
$loop = 0;
for ($loop = 0; $loop < $columns; $loop++) {
    $cycle_r[0] = $perm_r[$loop];
//print("\$loop: $loop ");
    $chain_r = array_merge($cycle_r, $subchain_r);
//print_r($chain[0]);
    $rows = count($chain_r);
    $cols = count($chain_r[0]);
//print("\$rows: $rows ");
    //$p = $chain;

    $rows = count($chain_r);
    $cols = count($chain_r[0]);

    $GJ_r = GaussJordanReduction($chain_r);

    $rows = count($GJ_r);
    $gj_cols = count($GJ_r[0]);
//print("\$cols: $cols, \$gj_cols: $gj_cols\n");
    $sum = 0;
    $inv_r = $GJ_r[0];
    for ($j = ($cols-1); $j < $gj_cols; $j++) {
        $k = $j-($cols-1);
        //print("$g_nodes[$k] : $inv[$j] \n");
        $sum = $inv_r[$j] + $sum;
    }

//print("Total : $sum \n");
if ($sum < $min_sum)
    $min_sum = $sum;
if ($sum > $max_sum)
    $max_sum = $sum;

for ($p = 1; $p < $rows ; $p++) {
    $sum = 0;
    $inv_r = $GJ_r[$p];

```

```

        for ($j = ($cols-1); $j < $gj_cols; $j++) {
            $sum = $inv_r[$j] + $sum;
        }
        $tot = $tot + $sum;
        $e = $p + 1;
        $sort_i = $i_sort[$p-1];
        //print("Row $p ($g_nodes[$sort_i]): $sum \n");
        //print("$sum\n");
    }
    $a = $tot/$rows;
    //print("Average: $a\n");

    // $a = 0; $tot = 0;
if (0 == 1) {
    // $chain = $p;
    $p = $chain;
    $I = IdentityMatrix(count($chain), count($chain[0]));
    $q = MatrixAdd($I, $chain);
    $w = 0;
    $s = 0;
    $e = array();
    while ($q[0][0] < 1.95) { // $GJ[0][$cols]
        $p = MatrixMultiply($p, $chain);
        $q = MatrixAdd($q, $p);
        for ($e=0; $e<count($q); $e++)
            $t[$e]=array_merge($p[$e], $q[$e]);
        print("\nMM:\n"); print_array($t);
        //for ($g=0; $g<count($q[0]); $g++)
        //    $s = $q[0][$g]+$s;
        //print("\$s: $s\n");
        $w=$w+1;
    }
    print("\$w: $w\n");
}

    } // end for loop

return $a;
}

/**** end of functions ****/

```

```

$A = array(
    array("a", "b", "c", "d"),
    array("e", "f", "h", "i"),
    array("j", "k", "l", "m"),
    array("n", "o", "p", "q")
);

//$minor = Minor($A, 2, 3);

//print_r($minor);

if ( 1 == 1 ) {

    $location[0] = "Dunster";
    //$model = "2012";
    $model = "default";

    $subchain = GenerateSubchain($location, $model, null);
    print_r($g_nodes);
    //print_r($subchain[18]);

}
else {

    $subchain = array (
        array(0, 0.5, 0.5, 0, 0, 0),
        array(0, 0, 0, 0, 0, 1),
        array(0, 0, 0, 0.5, 0.5, 0),
        array(0, 0, 0, 0, 0, 1),
        array(0, 0, 0, 0, 0, 1),
        array(0, 0, 0, 0, 0, 1)
    );

}

$rows = count($subchain);
$cols = count($subchain[0]);

    $GJ = GaussJordanReduction($subchain);
//print("\n\n");

```

```

print_array($GJ);

$rows = count($GJ);
$gj_cols = count($GJ[0]);
$sm[0] = 0.0; $i_holder[0] = 0;
for ($p = 0; $p < $rows ; $p++) {
    $sm[$p] = 0.0;
    $inv = $GJ[$p];
    for ($j = $cols-1; $j < $gj_cols; $j++) {
        //$k = $j-$cols;
        //print("$node_names[$k] : $inv[$j] \n");
        $sm[$p] = $inv[$j] + $sm[$p];
    }
    $tot = $tot + $sm[$p];
    $i_holder[$p] = $p;
    //$e = $p + 1;
print("$g_nodes[$p]: $sm[$p] \n");
    //print($sm[$p] . "\n");
}
// subtract one from $rows because of dummy holder at sm[0]
$act = $rows-1;
$avg = $tot/$act;
$tot = 0;
//print("\$avg: $avg\n");
exit;
}

$rows = count($subchain);
$cols = count($subchain[0]);

// we have a $rows x $cols array
// need to insert a first row to represent the distribution to the nodes

// first generate the first row using a Gaussian distribution

$sigma_sq = 0.0205;
//$sigma_sq = 0.035;
$mean = 0.5;

$value[0][0] = 0.0;

```

```

$cycle[0][0] = 0.0;
$perm[0][0] = 0.0;

$a = 2.0 * pi() * $sigma_sq;

$c = sqrt ($a);

$d = 1.0 / $c;

    // build a middle perm array
    // book-end it with zeros for Source and Sink
    for ($x = 0; $x < ($cols-2); $x++) {
        $step = $x/($cols-2);
        $e = -1 * pow(($step - $mean), 2);
        $f = 2 * $sigma_sq;
        $w = exp($e/$f);
        $value[0][$x] = $d * $w;
        //print("\$step: $step \$a: $a \$c: $c \$d: $d \$e: $e \$f $f \$value: $value[0][$x]
\n");

        //print("\$step: $step \$w: $w ");
        $sum = $sum + $value[0][$x];
    }

    // $value[0][$x] = 0.0;

    // normalize
    for ($x = 0; $x < ($cols-2); $x++) {
        $value[0][$x] = $value[0][$x]/$sum;
    }
print($value[0][$x] . "\n");
//print_r($value[0]);

    // generate the permutations of the Gaussian curve
    // each row of $perm is a shift of the Gaussian curve to the left

    // our first row
    $zero = array(0=>0);
    $temp2 = array_merge($zero, $value[0]);
    $perm[0] = array_merge($temp2, $zero);

    $interm[0] = $value[0];
    for ($y = 1; $y <= $rows; $y++) {
        $q = $y-1;

```



```

        $temp1 = $interm[$q];
//print("\$y: $y \$q: $q \$temp: ");
//print_r($temp1);
        $first = array_shift($temp1);
        array_push($temp1, $first);
        $interm[$y] = $temp1;

        $temp2 = array_merge($zero, $interm[$y]);
        $perm[$y] = array_merge($temp2, $zero);

//print_r($perm[$y]);
    }

//print_array($perm);

    // check our cumulative distribution function
    $cycle[0][0] = 0.0;
    $cycle[0] = $perm[0];
    $chain = array_merge($cycle, $subchain);
//print_array($chain);

    $rows = count($chain);
    $cols = count($chain[0]);

    $node_cdf = array(0);
    $node_cdf = array_pad($node_cdf, $rows, 0);

    $chain_cdf = array(array(0));
    $chain_cdf = array_pad($chain_cdf, $rows, $cols);

    for ($i=0; $i<$rows;$i++) {

        $cnt = count($chain[$i]);
        if ($cnt != $rows)
            print("row $i has $cnt columns, but there are $rows rows\n");

        $node_state = $chain[$i];
        $node_total = 0;
        for ($j = 0; $j < $cols; $j++) {
            $node_cdf[$j] = $node_total+$node_state[$j];
            $node_total = $node_cdf[$j];
        }
    }

```

```

if ($node_total < 0.99) {
    print("node $i has less than 0.99 total ($node_total)\n");
    print_r($chain[$i]);
    for ($k = 0; $k < $sz; $k++) {
        // print($node_names[$k] . ": " . $chain[$i][$k] . "\n");
    }
    exit;
}

if ($node_total > 1.01) {
    print("node $i has greater than 1.01 total ($node_total)\n");
    print_r($chain[$i]);
    for ($k = 0; $k < $sz; $k++) {
        // print($node_names[$k] . ": " . $chain[$i][$k] . "\n");
    }
    exit;
}

$chain_cdf[$i] = $node_cdf;
$node_cdf = array(0);

}

// sort our subchain by path length
// have to use first permutation
$GJ = GaussJordanReduction($chain);
//print("\n\n");
//print_array($GJ);

$rows = count($GJ);
$gj_cols = count($GJ[0]);
$sm[0] = 0.0; $i_holder[0] = 0;
for ($p = 0; $p < $rows; $p++) {
    $sm[$p] = 0.0;
    $inv = $GJ[$p];
    for ($j = $cols-1; $j < $gj_cols; $j++) {
        // $k = $j - $cols;
        //print("$node_names[$k] : $inv[$j] \n");
        $sm[$p] = $inv[$j] + $sm[$p];
    }
    $tot = $tot + $sm[$p];
    $i_holder[$p] = $p;
}

```

```

        // $e = $p + 1;
print("$g_nodes[$p]: $sm[$p] \n");
        // print($sm[$p] . "\n");
    }
    // subtract one from $rows because of dummy holder at sm[0]
    $act = $rows-1;
    $avg = $tot/$act;
    $tot = 0;
// print("\$avg: $avg\n");
// exit;

    // sort the array of sums and the indexes pointing to them
// print_r($sm); print_r($i_holder);
    $sm_t = $sm;
    // eliminate Source from indexes
    $sm_t = Array_Delete_Element($sm_t, 0);
    $i_holder = Array_Delete_Element($i_holder, 0);
    $bo = array_multisort($sm_t, $i_holder);
    // $sm_inc and i_inc will have increasing path length
    // $sm_t and $i_holder are used destructively
    $i_inc = $i_holder;
    $sm_inc = $sm_t;
// print_r($sm_t); print_r($i_holder);
// exit;

    // find the location of the average
    $i_avg = 0;
    while($sm_t[$i_avg] < $avg)
        $i_avg++;

    // find the node with path length closest to the average, add it to the indexes
    // delete the element, figure out high or low relative to average
    // add to indexes, repeat
    // $i_avg is the first sum over the average
    $i_count = 0; $h_count = 0;
    $sm_c = count($sm_t);
// print("\$i_avg: $i_avg\n");
    while (($sm_c > 0) && ($i_avg > -1)) {
        $sums[$i_count] = $sm_t[$i_avg];
        $i_sort[$i_count] = $i_holder[$i_avg];
// print("\$i_avg: $i_avg \$sums[\$i_count]: $sums[$i_count] \$i_count: $i_count ");
        $i_count++;
        $sm_t = Array_Delete_Element($sm_t, $i_avg);
        $i_holder = Array_Delete_Element($i_holder, $i_avg);
    }

```

```

        $c_avg = 0.0;
        for ($rr = 0; $rr < $i_count;$rr++)
            $c_avg = $c_avg+($sums[$rr]/$i_count);
        $sm_c = count($sm_t);
        if ((bccomp($c_avg, $avg, 16) > -1) || ($sm_c == $i_avg))
            $i_avg--;
    //print("\$i_avg: $i_avg \$c_avg: $c_avg \$avg: $avg \$sm_c $sm_c\n");
    //if ($i_avg == 3) {print_r($sm_t); exit;}
    }
    $avg = 0; $total = 0;

//print_r($sm_inc);print_r($i_inc);
//print_r($low); print_r($l_sum);
//print("\n");
//print("\$sums: ");
//print_r($sums);
//print("\n");

    // redistribute i_inc to be Gaussian/normal distribution
    $i_norm = array();
    $i_norm = array_pad($i_norm, sizeof($i_inc)+1, 0);
    $i_ninc = $i_inc;
    //$sm_norm = $sm_inc;
    $norm_c = count($i_ninc);
    if (($norm_c % 2) == 0) {
        $half = $norm_c/2;
        $even_c = 1;
        $odd_c = 1;
    } else {
        $half = round($norm_c/2, 1)+1;
        $even_c = 1;
        $odd_c = 1;
    }
//print_r($i_ninc);
//print("\$norm_c: $norm_c \$half: $half\n");
    $i_norm[$half] = $i_ninc[($norm_c-1)];
    $counter = 1;
    while ($counter < $norm_c) {
        if (($counter % 2) == 0) {
            $indx = $half-$even_c;
            $even_c++;
        } else {

```

```

        $indx = $half+$odd_c;
        $odd_c++;
    }
    $i_norm[$indx] = $i_ninc[($norm_c -1) - $counter];
    $counter++;
}

    // $i_temp = $zero+$i_norm;
    // $i_norm = $i_temp; //+$zero;
if (0 == 1) {
print("\n\n\n\n");
print_r($i_norm);
    for ($qwerty = 0; $qwerty < sizeof($i_norm); $qwerty++) {
        $node = $g_nodes[$i_norm[$qwerty]] ;
        $sum_norm = $sm[$i_norm[$qwerty]];
        //print("($qwerty): $node: $sum_norm \n");
        print("$sum_norm \n");
    }
}

}

print("\$sigma_sq: $sigma_sq\n");
// now find our minimum and maximum ranges based on uniform distribution
// $i_sort starts at 0, need buffer at 0
$name_sort[0] = "Source";
for ($see = 0; $see < sizeof($i_sort); $see++)
    $name_sort[$see+1] = $g_nodes[$i_sort[$see]];
$name_sort[$see+1] = "Sink";
//print_r($name_sort);
$subchain = GenerateSubchain($location, $model, $name_sort);

$max_sum = 0.0; $min_sum = 100000.0;
$avg_total = IterateOverSubchain($subchain, $perm, &$min_sum, &$max_sum);

    print("Uniform Distribution - Min Avg. Path Length: $min_sum    Max Avg. Path
Length: $max_sum\n");

// now find our minimum and maximum ranges based on normal value distribution
// $i_sort already contains Source
$name_sort[0] = "Source";

```

```

for ($ee = 0; $ee < sizeof($i_norm); $ee++)
    $name_sort[$ee] = $g_nodes[$i_norm[$ee]];
$name_sort[$ee] = "Sink";
//print_r($name_sort);
$subchain = GenerateSubchain($location, $model, $name_sort);

$max_sum = 0.0; $min_sum = 100000.0;
$avg_total = IterateOverSubchain($subchain, $perm, &$min_sum, &$max_sum);

print("Normal Distribution - Min Avg. Path Length: $min_sum    Max Avg. Path
Length: $max_sum\n");

//print("\$i_sort: ");print_r($i_sort);
//print("\$g_nodes: ");print_r($g_nodes);
// now find our minimum and maximum ranges based on random distribution
$real_min = 1000.0; $real_max = 0.0;
for ($rrnd = 0;$rrnd < 1; $rrnd++) {
    $i_rand = array();
    while (sizeof($i_rand) < sizeof($i_sort)) {
        $rand = mt_rand(1, sizeof($i_sort));
        if (! in_array($rand, $i_rand))
            $i_rand[] = $rand;
    }
//print("\$i_rand: ");print_r($i_rand);
$name_sort[0] = "Source";
for ($ee = 0; $ee < sizeof($i_rand); $ee++) {
    $name = $g_nodes[$i_rand[$ee]];
//print("\$ee: $ee, $name\n");
    $name_sort[$ee+1] = $name;
}
$name_sort[$ee+1] = "Sink";
//print_r($name_sort);
$subchain = GenerateSubchain($location, $model, $name_sort);

$max_sum = 0.0; $min_sum = 100000.0;
$avg_total = IterateOverSubchain($subchain, $perm, &$min_sum,
&$max_sum);

if ($max_sum > $real_max)
    $real_max = $max_sum;
if ($min_sum < $real_min)
    $real_min = $min_sum;

```

```

        if (($rrnd % 100) == 0)
            print("$rrnd...");

        if (($real_min < 2.0) || ($real_max > 3.0)) {
            print_r($name_sort);
            break;
        }
    }

    print("\nRandom Distribution - Min Avg. Path Length: $real_min   Max Avg. Path
Length: $real_max\n");

?>

sql.php

<?php

$sql = "postgreSQL";
//$sql = "mySQL";

if ($sql == "postgreSQL") {

    require_once "pgsql.php";

} elseif ($sql == "mySQL") {

    require_once "mysql.php";

} else {
    echo "Unknown SQL database setting, review sql.php.";
}

//$sqlserver = "";
$sqlserver='127.0.0.1';
//$sqlserver='localhost';

```

```

$sqluser='[REMOVED]';

$sqlpass='[REMOVED]';

$dbname = "sna_db";

$GLOBALS["db_conn"] = $sql_connect($sqlserver, $dbname, $sqluser, $sqlpass);

?>

sna_db database
--
-- PostgreSQL database dump
--

SET statement_timeout = 0;
SET client_encoding = 'UTF8';
SET standard_conforming_strings = off;
SET check_function_bodies = false;
SET client_min_messages = warning;
SET escape_string_warning = off;

SET search_path = public, pg_catalog;

--
-- Name: build_dl(integer[], character varying); Type: FUNCTION; Schema: public; Owner:
[REMOVED]
--

CREATE FUNCTION build_dl(integer[], character varying) RETURNS TABLE(dl text)
  LANGUAGE sql
  AS $_$

select src || ' ' || dest || ' ' || pct from markov_chain($1, $2);

$_$;

ALTER FUNCTION public.build_dl(integer[], character varying) OWNER TO [REMOVED];

--
-- Name: build_dl_node_list(integer[]); Type: FUNCTION; Schema: public; Owner: [RE-

```


MOVED]

--

```
CREATE FUNCTION build_dl_node_list(integer[]) RETURNS TABLE(dl text)
  LANGUAGE sql
  AS $_$
```

```
WITH dl_nodes(n) as
```

```
(
select (select array_to_string(array_agg(name), ', ')
from (select name from nodes where nodes.geo_loc in (select explode_array(cast($1 as integer[])))
order by name asc)
as n)
)
```

```
select 'Source, ' || dl_nodes.n || ', Sink' from dl_nodes
```

```
$_$;
```

```
ALTER FUNCTION public.build_dl_node_list(integer[]) OWNER TO [REMOVED];
```

--

```
-- Name: display_edge(integer); Type: FUNCTION; Schema: public; Owner: [REMOVED]
```

--

```
CREATE FUNCTION display_edge(uid integer) RETURNS TABLE(edge_uid integer, source_uid integer, source_name text, dest_uid integer, dest_name text, strength integer, model character varying)
  LANGUAGE sql
  AS $_$
```

```
select edges.uid as edge_uid,
```

```
na.uid as source_uid, COALESCE(na.name, '????') as source_name,
```

```
nb.uid as dest_uid, COALESCE(nb.name, '????') as dest_name,
```

```
strength, model from edges
```

```
left join nodes as na on edges.node_uid_a = na.uid left join nodes as nb on edges.node_uid_b = nb.uid
```

```
where edges.uid = $1
```

```
order by source_name, dest_name;
```

```
$_$;
```

```
ALTER FUNCTION public.display_edge(uid integer) OWNER TO [REMOVED];
```

```
--
```

```
-- Name: display_edges(integer); Type: FUNCTION; Schema: public; Owner: [REMOVED]
```

```
--
```

```
CREATE FUNCTION display_edges(uid integer) RETURNS TABLE(edge_uid integer, source_uid integer, source_name text, dest_uid integer, dest_name text, strength integer, model character varying)
```

```
    LANGUAGE sql
```

```
    AS $_$
```

```
select edges.uid as edge_uid,
```

```
na.uid as source_uid, COALESCE(na.name, '????') as source_name,
```

```
nb.uid as dest_uid, COALESCE(nb.name, '????') as dest_name,
```

```
strength, model from edges
```

```
left join nodes as na on edges.node_uid_a = na.uid left join nodes as nb on edges.node_uid_b = nb.uid
```

```
where edges.node_uid_a = $1 or edges.node_uid_b = $1
```

```
order by source_name, dest_name;
```

```
$_$;
```

```
ALTER FUNCTION public.display_edges(uid integer) OWNER TO [REMOVED];
```

```
--
```

```
-- Name: display_edges_for_dl(integer, integer); Type: FUNCTION; Schema: public; Owner: [REMOVED]
```

```
--
```

```
CREATE FUNCTION display_edges_for_dl(uid integer, rel_str integer) RETURNS TABLE(edge text)
```

```
    LANGUAGE sql
```

```
    AS $_$
```

```
select n1.name || ' ' || n2.name || ' ' || ((strength*$2)/(select sum(strength) from edges)) from edges left join nodes as n1 on node_uid_a = n1.uid left join nodes as n2 on node_uid_b =
```

```
n2.uid
```

```
$_$;
```

```
ALTER FUNCTION public.display_edges_for_dl(uid integer, rel_str integer) OWNER TO [REMOVED];
```

```
--
```

```
-- Name: display_geo_bin(integer); Type: FUNCTION; Schema: public; Owner: [REMOVED]
```

```
--
```

```
CREATE FUNCTION display_geo_bin(uid integer) RETURNS TABLE(uid integer, inner_loc text, outer_loc text)
```

```
    LANGUAGE sql
```

```
    AS $_$
```

```
select geo_bin.uid, i.name as inner, o.name as outer from geo_bin
left join geo_loc as i on i.uid = geo_bin.inner_geo
left join geo_loc as o on o.uid = geo_bin.outer_geo
where o.uid=$1;
```

```
$_$;
```

```
ALTER FUNCTION public.display_geo_bin(uid integer) OWNER TO [REMOVED];
```

```
--
```

```
-- Name: display_geo_bin(character varying); Type: FUNCTION; Schema: public; Owner: [REMOVED]
```

```
--
```

```
CREATE FUNCTION display_geo_bin(character varying) RETURNS TABLE(uid integer, inner_loc text, outer_loc text)
```

```
    LANGUAGE sql
```

```
    AS $_$
```

```
select geo_bin.uid, i.name as inner, o.name as outer from geo_bin
left join geo_loc as i on i.uid = geo_bin.inner_geo
left join geo_loc as o on o.uid = geo_bin.outer_geo
where o.name=$1;
```

```
$_$;
```

```
ALTER FUNCTION public.display_geo_bin(character varying) OWNER TO [REMOVED];
```

```
--
```

```
-- Name: display_geo_loc(character varying); Type: FUNCTION; Schema: public; Owner: [REMOVED]
```

```
--
```

```
CREATE FUNCTION display_geo_loc(character varying) RETURNS TABLE(uid integer)
  LANGUAGE sql
  AS $_$
```

```
select geo_loc.uid from geo_loc
where geo_loc.name = $1
order by geo_loc.name;
```

```
$_$;
```

```
ALTER FUNCTION public.display_geo_loc(character varying) OWNER TO [REMOVED];
```

```
--
```

```
-- Name: display_geo_loc(integer); Type: FUNCTION; Schema: public; Owner: [REMOVED]
```

```
--
```

```
CREATE FUNCTION display_geo_loc(integer) RETURNS TABLE(name character varying)
  LANGUAGE sql
  AS $_$
```

```
select geo_loc.name from geo_loc
where geo_loc.uid = $1
order by geo_loc.uid;
```

```
$_$;
```

```
ALTER FUNCTION public.display_geo_loc(integer) OWNER TO [REMOVED];
```

```
--
```

```
-- Name: display_geo_loc_uid(character varying); Type: FUNCTION; Schema: public; Owner:
```

[REMOVED]

--

```
CREATE FUNCTION display_geo_loc_uid(character varying) RETURNS TABLE(uid integer)
```

```
    LANGUAGE sql
```

```
    AS $_$
```

```
select uid from geo_loc where name=$1 order by uid asc;
```

```
$_$;
```

```
ALTER FUNCTION public.display_geo_loc_uid(character varying) OWNER TO [REMOVED];
```

--

```
-- Name: display_geo_nodes(integer); Type: FUNCTION; Schema: public; Owner: [REMOVED]
```

--

```
CREATE FUNCTION display_geo_nodes(uid integer) RETURNS TABLE(uid integer, name text)
```

```
    LANGUAGE sql
```

```
    AS $_$
```

```
select uid,name from nodes
```

```
where nodes.geo_loc = $1
```

```
order by name;
```

```
$_$;
```

```
ALTER FUNCTION public.display_geo_nodes(uid integer) OWNER TO [REMOVED];
```

--

```
-- Name: display_geo_nodes(character varying); Type: FUNCTION; Schema: public; Owner: [REMOVED]
```

--

```
CREATE FUNCTION display_geo_nodes(character varying) RETURNS TABLE(uid integer, name text)
```

```
    LANGUAGE sql
```

```
    AS $_$
```

```

select nodes.uid, nodes.name from nodes
left join geo_loc on nodes.geo_loc = geo_loc.uid
where geo_loc.name = $1
order by nodes.name;

```

```
$_ $;
```

```
ALTER FUNCTION public.display_geo_nodes(character varying) OWNER TO [REMOVED];
```

```

--
-- Name: display_inedges(integer); Type: FUNCTION; Schema: public; Owner: [REMOVED]
--

```

```

CREATE FUNCTION display_inedges(uid integer) RETURNS TABLE(edge_uid integer,
source_uid integer, source_name text, dest_uid integer, dest_name text, strength integer,
model character varying)

```

```
    LANGUAGE sql
```

```
    AS $_ $
```

```

select edges.uid as edge_uid, na.uid as source_uid, COALESCE(na.name, '????') as source_name,
nb.uid as dest_uid, nb.name as dest_name,
strength, model from edges
left join nodes as na on edges.node_uid_a = na.uid left join nodes as nb on edges.node_uid_b
= nb.uid
where edges.node_uid_b = $1
order by source_name, dest_name;

```

```
$_ $;
```

```
ALTER FUNCTION public.display_inedges(uid integer) OWNER TO [REMOVED];
```

```

--
-- Name: display_names(integer); Type: FUNCTION; Schema: public; Owner: [REMOVED]
--

```

```

CREATE FUNCTION display_names(integer) RETURNS TABLE(name character varying)

```

```
    LANGUAGE sql
```

```
    AS $_ $
```

```
select name from nodes where uid=$1 order by uid asc;
```

```
$_$;
```

```
ALTER FUNCTION public.display_names(integer) OWNER TO [REMOVED];
```

```
--
```

```
-- Name: display_node(integer); Type: FUNCTION; Schema: public; Owner: [REMOVED]
```

```
--
```

```
CREATE FUNCTION display_node(integer) RETURNS TABLE(uid integer, name character
varying, address1 text, address2 text, geo_loc character varying, industry character varying)
```

```
LANGUAGE sql
```

```
AS $_$
```

```
select nodes.uid as uid, nodes.name as name, nodes.address1 as address1,
nodes.address2 as address2, geo_loc.name as geo_loc, nodes.industry as industry
from nodes left join geo_loc on geo_loc.uid = nodes.geo_loc
where nodes.uid=$1;
```

```
$_$;
```

```
ALTER FUNCTION public.display_node(integer) OWNER TO [REMOVED];
```

```
--
```

```
-- Name: display_node_uid(character varying); Type: FUNCTION; Schema: public; Owner:
[REMOVED]
```

```
--
```

```
CREATE FUNCTION display_node_uid(character varying) RETURNS TABLE(uid integer)
```

```
LANGUAGE sql
```

```
AS $_$
```

```
select uid from nodes where name=$1 order by uid asc;
```

```
$_$;
```

```
ALTER FUNCTION public.display_node_uid(character varying) OWNER TO [REMOVED];
```

```
--
```

```
-- Name: display_outedges(integer); Type: FUNCTION; Schema: public; Owner: [REMOVED]
```

```
--
```

```
CREATE FUNCTION display_outedges(uid integer) RETURNS TABLE(edge_uid integer,
source_uid integer, source_name text, dest_uid integer, dest_name text, strength integer,
```

```

model character varying)
  LANGUAGE sql
  AS $_$

select edges.uid as edge_uid, na.uid as source_uid, na.name as source_name,
nb.uid as dest_uid, COALESCE(nb.name, '????') as dest_name,
strength, model from edges
left join nodes as na on edges.node_uid_a = na.uid left join nodes as nb on edges.node_uid_b
= nb.uid
where edges.node_uid_a = $1
order by source_name, dest_name;

$_$;

ALTER FUNCTION public.display_outedges(uid integer) OWNER TO [REMOVED];

--
-- Name: explode_array(anyarray); Type: FUNCTION; Schema: public; Owner: [REMOVED]
--

CREATE FUNCTION explode_array(in_array anyarray) RETURNS SETOF anyelement
  LANGUAGE sql IMMUTABLE
  AS $_$ select ($1)[s] from generate_series(1,array_upper($1, 1)) as s; $_$;

ALTER FUNCTION public.explode_array(in_array anyarray) OWNER TO [REMOVED];

--
-- Name: geo_chain(integer); Type: FUNCTION; Schema: public; Owner: [REMOVED]
--

CREATE FUNCTION geo_chain(integer) RETURNS TABLE(uid integer, outer_geo integer,
outer_geo_name character varying, inner_geo integer, inner_geo_name character varying)
  LANGUAGE sql
  AS $_$
WITH RECURSIVE chain(uid, outer_geo, inner_geo) AS

(SELECT geo_bin.uid as uid, geo_bin.outer_geo as outer_geo, geo_bin.inner_geo as in-
ner_geo from geo_bin
WHERE geo_bin.inner_geo=$1
UNION ALL SELECT

```



```

geo_bin.uid, geo_bin.outer_geo as outer_geo, geo_bin.inner_geo as inner_geo from chain,
geo_bin
WHERE geo_bin.inner_geo = chain.outer_geo)

```

```

SELECT chain.uid, o.uid as outer_geo, o.name as outer_geo_name, i.uid as inner_geo, i.name
as inner_geo_name from chain
left join geo_loc as o on o.uid=chain.outer_geo
left join geo_loc as i on i.uid=chain.inner_geo;

```

```

$_$;

```

```

ALTER FUNCTION public.geo_chain(integer) OWNER TO [REMOVED];

```

```

--

```

```

-- Name: geo_chain(character varying); Type: FUNCTION; Schema: public; Owner: [RE-
MOVED]

```

```

--

```

```

CREATE FUNCTION geo_chain(character varying) RETURNS TABLE(uid integer,
outer_geo integer, outer_geo_name character varying, inner_geo integer, inner_geo_name
character varying)

```

```

    LANGUAGE sql

```

```

    AS $_$

```

```

WITH RECURSIVE chain(uid, outer_geo, inner_geo) AS

```

```

(SELECT geo_bin.uid, geo_bin.outer_geo as outer_geo, geo_bin.inner_geo as inner_geo from
geo_bin

```

```

left join geo_loc on geo_loc.uid=geo_bin.inner_geo

```

```

WHERE geo_loc.name=$1

```

```

UNION ALL SELECT

```

```

geo_bin.uid, geo_bin.outer_geo as outer_geo, geo_bin.inner_geo as inner_geo from chain,
geo_bin

```

```

WHERE geo_bin.inner_geo = chain.outer_geo)

```

```

SELECT chain.uid, o.uid as outer_geo, o.name as outer_geo_name, i.uid as inner_geo, i.name
as inner_geo_name from chain

```

```

left join geo_loc as o on o.uid=chain.outer_geo

```

```

left join geo_loc as i on i.uid=chain.inner_geo;

```

```

$_$;

```

```

ALTER FUNCTION public.geo_chain(character varying) OWNER TO [REMOVED];

--
-- Name: markov_chain(integer[], character varying); Type: FUNCTION; Schema: public;
Owner: [REMOVED]
--

CREATE FUNCTION markov_chain(integer[], character varying) RETURNS TABLE(src_uid
integer, src text, dest text, str bigint, total bigint, pct double precision)
    LANGUAGE sql
    AS $_$
WITH strengths(node_uid, total) AS
(
    SELECT * from sum_strengths($1, $2)
)
-- then do the percentages to each dest node from source node
(
SELECT edges.node_uid_a as src_uid,
COALESCE(source.name, 'Source') as src,
COALESCE(dest.name, 'Sink') as dst,
sum(edges.strength) as str,
strengths.total as total,
(sum(edges.strength)::float)/strengths.total::float as pct
from edges
left join nodes as source on source.uid=edges.node_uid_a
left join nodes as dest on dest.uid=edges.node_uid_b
left join strengths on edges.node_uid_a = strengths.node_uid
where source.geo_loc in (select explode_array($1)) and dest.geo_loc in (select explode_ar-
ray($1)) and edges.model=$2
group by edges.node_uid_b, src, dst, strengths.total, edges.node_uid_a
)
UNION
-- the percentages from Source to the destination nodes
--(
--SELECT 0 as src_uid,
--'Source' as src,
--dest.name as dst,
--sum(edges.strength) as str,
--strengths.total as total,
--(sum(edges.strength)::float)/strengths.total::float as pct

```

```

--from edges
--left join nodes as source on edges.node_uid_a=source.uid
--left join nodes as dest on edges.node_uid_b=dest.uid
--left join strengths on strengths.node_uid=0
--where source.geo_loc not in (select explode_array($1)) and dest.geo_loc in (select ex-
plode_array($1)) and edges.model=$2
--group by edges.node_uid_b, src, dst, strengths.total
--)

(
select * from source_strengths($1)
)

UNION
-- the percentages from a local node to a Sink node
(
SELECT edges.node_uid_a as src_uid,
source.name as src,
'Sink' as dst,
sum(edges.strength) as str,
strengths.total as total,
(sum(edges.strength)::float)/strengths.total::float as pct
from edges
left join nodes as source on edges.node_uid_a=source.uid
left join nodes as dest on edges.node_uid_b=dest.uid
left join strengths on edges.node_uid_a = strengths.node_uid
where source.geo_loc in (select explode_array($1)) and dest.geo_loc not in (select explode_ar-
ray($1)) and edges.model=$2
group by edges.node_uid_a, src, dst, strengths.total
)

UNION
-- create entries for connected local nodes that have not been surveyed - assume 100% leaves the
community
(
SELECT nodes.uid as src_uid,
nodes.name as src,
'Sink' as dst,
1 as str,
1 as total,
1 as pct
from nodes
where nodes.uid in

```

```

        ((select distinct(node_uid_b) from edges
        left join nodes on edges.node_uid_b=nodes.uid
        where nodes.geo_loc in (select explode_array($1)))
        EXCEPT
        (select distinct(node_uid_a) from edges
        left join nodes on edges.node_uid_a=nodes.uid
        where nodes.geo_loc in (select explode_array($1)) and edges.model=$2)
        )
    )

UNION
-- create entries for identified businesses that are not connected (orphans)
SELECT nodes.uid as src_uid,
nodes.name as src,
'Sink' as dst,
1 as str,
1 as total,
1 as pct
from nodes
where nodes.uid in
(
    select node_uid from return_orphan_nodes($1, $2)
)

order by src_uid, src, dst;

$_$;

ALTER FUNCTION public.markov_chain(integer[], character varying) OWNER TO [REMOVED];

--
-- Name: mc_output(integer[], character varying); Type: FUNCTION; Schema: public; Owner: [REMOVED]
--

CREATE FUNCTION mc_output(integer[], character varying) RETURNS TABLE(src text,
dest text, pct double precision)
    LANGUAGE sql
    AS $_$

```

```
select src, dest, pct from markov_chain($1, $2);
```

```
$_$;
```

```
ALTER FUNCTION public.mc_output(integer[], character varying) OWNER TO [REMOVED];
```

```
--
```

```
-- Name: return_orphan_nodes(integer[], character varying); Type: FUNCTION; Schema: public; Owner: [REMOVED]
```

```
--
```

```
CREATE FUNCTION return_orphan_nodes(integer[], character varying) RETURNS TABLE(node_uid integer)
```

```
LANGUAGE sql
```

```
AS $_$
```

```
select uid as node_uid from nodes where geo_loc in (select * from explode_array($1))
```

```
EXCEPT
```

```
(
  select uid from nodes where uid in
  (
    select node_uid as n_a from sum_exit_nodes_strengths($1, $2)
    UNION
    select node_uid as n_b from sum_edge_strengths($1, $2)
  )
)
```

```
$_$;
```

```
ALTER FUNCTION public.return_orphan_nodes(integer[], character varying) OWNER TO [REMOVED];
```

```
--
```

```
-- Name: source_strengths(integer[]); Type: FUNCTION; Schema: public; Owner: [REMOVED]
```

```
--
```

```
CREATE FUNCTION source_strengths(integer[]) RETURNS TABLE(src_uid integer, src
text, dest text, str bigint, total bigint, pct double precision)
LANGUAGE sql
AS $_$
```

```
WITH strengths(total, per_node) AS
(
    SELECT count(*) as total, (1::float/count(*)::float) as per_node from nodes
        where nodes.geo_loc in (select explode_array($1))
)
```

```
SELECT 0 as src_uid,
cast('Source' as text) as src,
nodes.name as dst,
cast(1 as bigint) as str,
strengths.total as total,
strengths.per_node as pct
from nodes
right join strengths on 1 = 1
where nodes.geo_loc in (select explode_array($1))

$_$;
```

```
ALTER FUNCTION public.source_strengths(integer[]) OWNER TO [REMOVED];
```

```
--
-- Name: sum_edge_strengths(integer[], character varying); Type: FUNCTION; Schema: public;
Owner: [REMOVED]
--
```

```
CREATE FUNCTION sum_edge_strengths(integer[], character varying) RETURNS TA-
BLE(node_uid integer, total bigint)
LANGUAGE sql
AS $_$
(select nodes.uid as node_uid, COALESCE(sum(edges.strength), 0) as total from edges
left join nodes on edges.node_uid_a=nodes.uid
where nodes.geo_loc in (select explode_array($1)) and edges.model=$2
group by nodes.uid)

$_$;
```

```
ALTER FUNCTION public.sum_edge_strengths(integer[], character varying) OWNER TO
[REMOVED];
```

```
--
-- Name: sum_exit_nodes_strengths(integer[], character varying); Type: FUNCTION; Schema:
public; Owner: [REMOVED]
```

```
CREATE FUNCTION sum_exit_nodes_strengths(integer[], character varying) RETURNS
TABLE(node_uid integer, total bigint)
LANGUAGE sql
AS $_$
```

```
select nodes.uid, COALESCE(sum(src.strength), 1) as total from nodes
left join edges as src on src.node_uid_a = nodes.uid
left join edges as dest on dest.node_uid_b = nodes.uid
where nodes.uid in
(
(
select distinct(node_uid_b) from edges
left join nodes on edges.node_uid_b=nodes.uid
where nodes.geo_loc in (select explode_array($1))
) EXCEPT (
select node_uid from sum_edge_strengths($1, $2)
)
)
)
group by nodes.uid
```

```
$_$;
```

```
ALTER FUNCTION public.sum_exit_nodes_strengths(integer[], character varying) OWNER
TO [REMOVED];
```

```
--
-- Name: sum_orphan_nodes_strengths(integer[], character varying); Type: FUNCTION;
Schema: public; Owner: [REMOVED]
```

```

--

CREATE FUNCTION sum_orphan_nodes_strengths(integer[], character varying) RETURNS
TABLE(node_uid integer, total bigint)
  LANGUAGE sql
  AS $_$

select uid as node_uid, count(uid) as total from nodes where nodes.uid in
(
  select * from return_orphan_nodes($1, $2)
)
group by uid

$_$;

ALTER FUNCTION public.sum_orphan_nodes_strengths(integer[], character varying)
OWNER TO [REMOVED];

--
-- Name: sum_source_strengths(integer[], character varying); Type: FUNCTION; Schema: pub-
lic; Owner: [REMOVED]
--

CREATE FUNCTION sum_source_strengths(integer[], character varying) RETURNS TA-
BLE(node_uid integer, total bigint)
  LANGUAGE sql
  AS $_$

SELECT 0 as node_uid, count(*) as total from nodes where nodes.geo_loc in (select ex-
plode_array($1))
group by 1

$_$;

ALTER FUNCTION public.sum_source_strengths(integer[], character varying) OWNER TO
[REMOVED];

--
-- Name: sum_strengths(integer[], character varying); Type: FUNCTION; Schema: public;
Owner: [REMOVED]

```



```

--

CREATE FUNCTION sum_strengths(integer[], character varying) RETURNS TA-
BLE(node_uid integer, total bigint)
  LANGUAGE sql
  AS $_$

-- develop the totals from each node
(
select node_uid, total from sum_edge_strengths($1, $2)

UNION
-- then the distribution from "Source" beyond the local area

select node_uid, total from sum_source_strengths($1, $2)

UNION
-- do nodes that were connected by not surveyed

select node_uid, total from sum_exit_nodes_strengths($1, $2)

UNION
-- and last the nodes that have no edges to other nodes - assume 100% leaves economy

select node_uid, total from sum_orphan_nodes_strengths($1, $2)
)

order by node_uid
$_$;

ALTER FUNCTION public.sum_strengths(integer[], character varying) OWNER TO [RE-
MOVED];

--
-- Name: test(character varying); Type: FUNCTION; Schema: public; Owner: [REMOVED]
--

CREATE FUNCTION test(character varying) RETURNS TABLE(string character varying)
  LANGUAGE sql
  AS $_$
select $1 from edges;

```

```
$_$;

ALTER FUNCTION public.test(character varying) OWNER TO [REMOVED];

SET default_tablespace = '';

SET default_with_oids = false;

--
-- Name: edges; Type: TABLE; Schema: public; Owner: [REMOVED]; Tablespace:
--

CREATE TABLE edges (
    uid integer NOT NULL,
    node_uid_a integer,
    node_uid_b integer,
    strength integer,
    model character varying(80)
);

ALTER TABLE public.edges OWNER TO [REMOVED];

--
-- Name: edges_uid_seq; Type: SEQUENCE; Schema: public; Owner: [REMOVED]
--

CREATE SEQUENCE edges_uid_seq
    START WITH 1
    INCREMENT BY 1
    NO MAXVALUE
    NO MINVALUE
    CACHE 1;

ALTER TABLE public.edges_uid_seq OWNER TO [REMOVED];

--
-- Name: edges_uid_seq; Type: SEQUENCE OWNED BY; Schema: public; Owner: [REMOVED]
--
```

```
ALTER SEQUENCE edges_uid_seq OWNED BY edges.uid;

--
-- Name: edges_uid_seq; Type: SEQUENCE SET; Schema: public; Owner: [REMOVED]
--

SELECT pg_catalog.setval('edges_uid_seq', 590, true);

--
-- Name: geo_bin; Type: TABLE; Schema: public; Owner: [REMOVED]; Tablespace:
--

CREATE TABLE geo_bin (
    uid integer NOT NULL,
    outer_geo integer,
    inner_geo integer
);

ALTER TABLE public.geo_bin OWNER TO [REMOVED];

--
-- Name: geo_bin_uid_seq; Type: SEQUENCE; Schema: public; Owner: [REMOVED]
--

CREATE SEQUENCE geo_bin_uid_seq
    START WITH 1
    INCREMENT BY 1
    NO MAXVALUE
    NO MINVALUE
    CACHE 1;

ALTER TABLE public.geo_bin_uid_seq OWNER TO [REMOVED];

--
-- Name: geo_bin_uid_seq; Type: SEQUENCE OWNED BY; Schema: public; Owner: [REMOVED]
--
```

```
ALTER SEQUENCE geo_bin_uid_seq OWNED BY geo_bin.uid;

--
-- Name: geo_bin_uid_seq; Type: SEQUENCE SET; Schema: public; Owner: [REMOVED]
--

SELECT pg_catalog.setval('geo_bin_uid_seq', 100, true);

--
-- Name: geo_loc; Type: TABLE; Schema: public; Owner: [REMOVED]; Tablespace:
--

CREATE TABLE geo_loc (
    uid integer NOT NULL,
    name character varying(80)
);

ALTER TABLE public.geo_loc OWNER TO [REMOVED];

--
-- Name: geo_loc_uid_seq; Type: SEQUENCE; Schema: public; Owner: [REMOVED]
--

CREATE SEQUENCE geo_loc_uid_seq
    START WITH 1
    INCREMENT BY 1
    NO MAXVALUE
    NO MINVALUE
    CACHE 1;

ALTER TABLE public.geo_loc_uid_seq OWNER TO [REMOVED];

--
-- Name: geo_loc_uid_seq; Type: SEQUENCE OWNED BY; Schema: public; Owner: [REMOVED]
--
```

```
ALTER SEQUENCE geo_loc_uid_seq OWNED BY geo_loc.uid;
```

```
--  
-- Name: geo_loc_uid_seq; Type: SEQUENCE SET; Schema: public; Owner: [REMOVED]  
--
```

```
SELECT pg_catalog.setval('geo_loc_uid_seq', 108, true);
```

```
--  
-- Name: nodes; Type: TABLE; Schema: public; Owner: [REMOVED]; Tablespace:  
--
```

```
CREATE TABLE nodes (  
    uid integer NOT NULL,  
    name character varying(18),  
    address1 character varying(80),  
    address2 character varying(80),  
    geo_loc integer,  
    industry character varying(80)  
);
```

```
ALTER TABLE public.nodes OWNER TO [REMOVED];
```

```
--  
-- Name: nodes_uid_seq; Type: SEQUENCE; Schema: public; Owner: [REMOVED]  
--
```

```
CREATE SEQUENCE nodes_uid_seq  
    START WITH 1  
    INCREMENT BY 1  
    NO MAXVALUE  
    NO MINVALUE  
    CACHE 1;
```

```
ALTER TABLE public.nodes_uid_seq OWNER TO [REMOVED];
```

```
--  
-- Name: nodes_uid_seq; Type: SEQUENCE OWNED BY; Schema: public; Owner: [RE-
```

```

MOVED]
--

ALTER SEQUENCE nodes_uid_seq OWNED BY nodes.uid;

--
-- Name: nodes_uid_seq; Type: SEQUENCE SET; Schema: public; Owner: [REMOVED]
--

SELECT pg_catalog.setval('nodes_uid_seq', 220, true);

--
-- Name: uid; Type: DEFAULT; Schema: public; Owner: [REMOVED]
--

ALTER TABLE edges ALTER COLUMN uid SET DEFAULT
nextval('edges_uid_seq'::regclass);

--
-- Name: uid; Type: DEFAULT; Schema: public; Owner: [REMOVED]
--

ALTER TABLE geo_bin ALTER COLUMN uid SET DEFAULT
nextval('geo_bin_uid_seq'::regclass);

--
-- Name: uid; Type: DEFAULT; Schema: public; Owner: [REMOVED]
--

ALTER TABLE geo_loc ALTER COLUMN uid SET DEFAULT
nextval('geo_loc_uid_seq'::regclass);

--
-- Name: uid; Type: DEFAULT; Schema: public; Owner: [REMOVED]
--

ALTER TABLE nodes ALTER COLUMN uid SET DEFAULT

```

```
nextval('nodes_uid_seq'::regclass);
```

```
--
```

```
-- Data for Name: edges; Type: TABLE DATA; Schema: public; Owner: [REMOVED]
```

```
--
```

```
COPY edges (uid, node_uid_a, node_uid_b, strength, model) FROM stdin;
```

```
186 119 116 100 default
189 119 123 466 default
190 119 124 58 default
191 119 120 100 default
192 119 127 100 default
188 119 122 175 default
193 117 128 70 default
78 60 211 1 2010
61 14 211 1 2010
70 96 60 1 2010
88 74 9 1 2010
89 74 57 1 2010
90 74 8 1 2010
91 70 74 1 2010
92 30 74 1 2010
93 42 74 1 2010
97 33 42 1 2010
4 11 14 1 2010
5 2 14 1 2010
6 14 2 1 2010
7 14 11 1 2010
8 14 12 1 2010
9 1 12 1 2010
10 1 9 1 2010
11 1 25 1 2010
194 117 128 70 2007
12 1 29 1 2010
28 30 15 1 2010
29 31 15 1 2010
113 32 47 1 2010
34 41 52 1 2010
62 14 60 1 2010
63 14 61 1 2010
64 14 62 1 2010
```

66	14	64	1	2010
67	14	65	1	2010
71	10	60	1	2010
72	51	60	1	2010
73	38	60	1	2010
74	32	60	1	2010
75	60	28	1	2010
76	60	25	1	2010
77	60	21	1	2010
201	117	129	30	default
197	117	127	110	2007
79	69	28	1	2010
80	28	69	1	2010
81	12	70	1	2010
82	69	12	1	2010
30	51	52	1	2010
124	51	56	1	2010
32	52	12	1	2010
83	32	55	1	2010
68	14	59	1	2010
85	14	71	1	2010
65	14	73	1	2010
69	14	72	1	2010
125	51	75	1	2010
126	51	69	1	2010
127	51	76	1	2010
128	51	77	1	2010
131	51	10	1	2010
132	51	78	1	2010
133	51	79	1	2010
134	51	80	1	2010
135	51	29	1	2010
136	51	28	1	2010
137	51	25	1	2010
138	51	21	1	2010
142	42	51	1	2010
143	32	51	1	2010
144	33	51	1	2010
145	51	12	1	2010
147	56	12	1	2010
148	60	12	1	2010
152	75	12	1	2010

153	76	12	1	2010
156	8	56	1	2010
157	8	59	1	2010
158	8	60	1	2010
159	8	75	1	2010
196	117	116	300	2007
160	8	69	1	2010
164	32	8	1	2010
165	42	8	1	2010
166	34	8	1	2010
168	8	29	1	2010
169	10	12	1	2010
170	34	10	1	2010
15	84	1	1	2010
17	82	1	1	2010
18	85	1	1	2010
13	81	1	1	2010
171	88	1	1	2010
172	87	1	1	2010
173	83	1	1	2010
174	86	1	1	2010
177	94	51	1	2010
178	95	51	1	2010
195	117	116	500	default
198	117	139	50	default
199	117	139	50	2007
200	117	129	70	2007
202	117	130	30	2007
204	117	9	20	2007
205	117	9	10	default
206	117	4	50	2007
207	117	4	50	default
208	117	131	50	2007
209	117	131	50	default
210	117	138	100	2007
211	117	138	20	default
212	117	132	50	2007
213	117	132	50	default
214	117	137	70	2007
215	117	137	30	default
216	117	133	30	2007
217	117	133	30	default

218	117	136	30	default
219	117	134	30	default
220	117	135	40	default
51	55	2	2	2007
52	55	25	3	default
162	8	211	1	2010
48	55	169	87	2007
49	55	2	6	default
163	8	211	1	2010
161	8	211	1	2010
33	52	211	1	2010
150	56	211	1	2010
154	76	211	1	2010
155	77	211	1	2010
151	75	211	1	2010
87	14	211	1	2010
141	51	211	1	2010
129	51	211	1	2010
139	51	211	1	2010
146	51	211	1	2010
140	51	211	1	2010
27	15	211	1	2010
223	145	148	190	default
224	145	149	63	default
225	145	21	62	default
226	145	25	22	default
227	145	28	10	default
228	145	143	19	default
229	145	116	109	default
230	145	144	38	default
231	145	140	22	default
232	145	92	52	default
233	145	150	52	default
234	145	128	19	default
235	145	141	8	default
236	145	151	8	default
203	117	130	10	default
237	152	116	210	2007
239	152	116	260	default
240	152	139	8	default
238	152	139	20	2007
242	152	155	225	default

241	152	155	148	2007
243	152	59	30	2007
244	152	59	37	default
245	152	156	40	2007
246	152	156	72	default
247	152	157	39	2007
248	152	157	100	default
249	152	98	73	2007
251	152	98	51	default
252	152	158	51	default
250	152	158	74	2007
253	152	144	17	2007
254	152	144	12	default
255	152	21	12	2007
256	152	21	29	default
257	152	159	44	2007
258	152	128	18	2007
259	152	128	100	default
260	152	164	17	default
261	152	160	9	2007
262	152	160	25	default
263	152	133	4	default
264	152	25	4	default
265	152	9	4	default
266	152	9	40	2007
267	152	153	14	2007
268	152	154	30	2007
269	152	163	27	2007
270	152	161	90	2007
271	152	132	45	2007
272	152	162	18	2007
273	139	116	90	default
274	139	9	1	default
275	139	87	4	default
276	139	88	1	default
277	139	83	678	default
278	139	165	226	default
279	166	21	195	default
280	166	167	195	default
281	166	168	50	default
282	166	169	50	default
284	166	171	293	default

285	166	144	44	default
286	166	172	98	default
287	166	139	10	default
288	166	9	10	default
289	166	173	25	default
283	166	122	25	default
290	166	92	1	default
291	174	98	200	default
292	174	133	50	default
293	174	178	50	default
294	174	139	50	default
295	174	176	150	default
41	36	55	1	alt
14	83	1	1	alt
296	174	9	100	default
297	174	59	100	default
299	174	116	250	default
298	174	184	50	default
300	188	116	1000	default
301	189	116	360	2007
302	189	116	300	default
303	189	190	250	2007
304	189	190	280	default
305	189	191	50	2007
306	189	191	60	default
307	189	98	50	2007
308	189	98	60	default
309	189	9	50	2007
35	42	52	1	2010
37	32	52	1	2010
36	8	52	1	2010
38	8	14	1	2010
39	10	14	1	2010
40	32	14	1	2010
114	33	34	1	2010
42	35	55	1	2010
43	34	55	1	2010
44	33	55	1	2010
115	34	33	1	2010
116	34	35	1	2010
117	35	34	1	2010
118	35	36	1	2010

119	36	35	1	2010
120	51	14	1	2010
26	15	211	1	2010
56	14	211	1	2010
59	14	25	1	2010
60	14	29	1	2010
310	189	9	80	default
311	189	139	90	2007
312	189	139	70	default
313	13	195	76	2007
314	13	192	380	2007
316	13	12	345	2007
317	13	128	54	2007
321	13	197	15	default
320	13	197	145	2007
332	55	116	210	default
324	55	9	164	2007
327	55	199	522	2007
330	55	198	16	2007
325	55	194	126	default
333	55	124	210	2007
328	55	199	581	default
329	55	9	26	default
334	189	128	50	default
331	55	198	10	default
326	55	169	34	default
322	13	193	363	default
319	13	128	610	default
323	13	196	14	default
50	55	22	4	default
335	189	128	50	2007
336	189	127	100	default
337	189	127	100	2007
338	145	142	62	default
339	145	139	139	default
340	145	9	124	default
57	14	211	1	2010
342	200	187	10	default
343	200	139	4	default
347	200	205	9	default
348	200	86	2	default
349	200	21	8	default

350	200	206	4	default
351	200	9	4	default
352	200	127	5	default
353	200	207	20	default
354	200	208	8	default
355	200	209	8	default
356	200	198	4	default
357	200	197	4	default
358	200	210	2	default
403	116	211	750	default
359	200	211	4	default
360	200	92	2	default
361	200	88	1	default
362	200	168	10	default
363	200	144	4	default
364	200	167	3	default
365	200	213	4	default
346	200	204	58	default
345	200	203	58	default
341	200	116	756	default
344	200	201	11	default
366	52	124	280	2007
367	52	124	264	default
368	52	25	7	2007
369	52	25	4	default
370	52	9	4	2007
371	52	59	131	default
374	52	81	50	2007
375	52	81	11	default
376	52	215	27	2007
377	52	216	20	2007
378	52	216	18	default
379	52	214	7	2007
380	52	217	50	2007
381	52	1	189	2007
382	52	1	5	default
383	52	218	17	2007
384	52	51	16	2007
385	52	51	17	default
386	52	69	24	default
387	52	219	39	default
388	52	220	25	default

389	52	127	59	2007
390	52	127	77	default
391	52	29	32	2007
392	52	29	89	default
393	52	168	5	2007
394	52	168	12	default
395	52	144	17	2007
396	52	197	4	2007
397	52	197	3	default
399	52	52	15	2007
400	52	83	17	default
401	52	83	15	2007
372	52	92	201	2007
373	52	92	238	default
402	116	139	150	default
404	119	123	466	2012
405	119	124	58	2012
406	119	120	100	2012
407	119	127	100	2012
408	119	122	175	2012
409	117	128	70	2012
410	117	129	30	2012
411	117	116	500	2012
412	117	139	50	2012
413	117	9	10	2012
414	117	4	50	2012
415	117	131	50	2012
416	117	138	20	2012
417	117	132	50	2012
418	117	137	30	2012
419	117	133	30	2012
420	117	136	30	2012
421	117	134	30	2012
422	117	135	40	2012
423	55	25	3	2012
424	55	2	6	2012
425	145	148	190	2012
426	145	149	63	2012
427	145	21	62	2012
428	145	25	22	2012
429	145	28	10	2012
430	145	143	19	2012

431	145	116	109	2012
432	145	144	38	2012
433	145	140	22	2012
434	145	92	52	2012
435	145	150	52	2012
436	145	128	19	2012
437	145	141	8	2012
438	145	151	8	2012
439	117	130	10	2012
440	152	116	260	2012
441	152	139	8	2012
442	152	155	225	2012
443	152	59	37	2012
444	152	156	72	2012
445	152	157	100	2012
446	152	98	51	2012
447	152	158	51	2012
448	152	144	12	2012
449	152	21	29	2012
450	152	128	100	2012
451	152	164	17	2012
452	152	160	25	2012
453	152	133	4	2012
454	152	25	4	2012
455	152	9	4	2012
456	139	116	90	2012
457	139	9	1	2012
458	139	87	4	2012
459	139	88	1	2012
460	139	83	678	2012
461	139	165	226	2012
462	166	21	195	2012
463	166	167	195	2012
464	166	168	50	2012
465	166	169	50	2012
466	166	171	293	2012
467	166	144	44	2012
468	166	172	98	2012
469	166	139	10	2012
470	166	9	10	2012
471	166	173	25	2012
472	166	122	25	2012

473	166	92	1	2012
474	174	98	200	2012
475	174	133	50	2012
476	174	178	50	2012
477	174	139	50	2012
478	174	176	150	2012
479	174	9	100	2012
480	174	59	100	2012
481	174	116	250	2012
482	174	184	50	2012
483	188	116	1000	2012
484	189	116	300	2012
485	189	190	280	2012
486	189	191	60	2012
487	189	98	60	2012
488	189	9	80	2012
489	189	139	70	2012
490	13	197	15	2012
491	55	116	210	2012
492	55	194	126	2012
493	55	199	581	2012
494	55	9	26	2012
495	189	128	50	2012
496	55	198	10	2012
497	55	169	34	2012
501	55	22	4	2012
502	189	127	100	2012
503	145	142	62	2012
504	145	139	139	2012
505	145	9	124	2012
506	200	187	10	2012
507	200	139	4	2012
508	200	205	9	2012
509	200	86	2	2012
510	200	21	8	2012
511	200	206	4	2012
512	200	9	4	2012
513	200	127	5	2012
514	200	207	20	2012
515	200	208	8	2012
516	200	209	8	2012
517	200	198	4	2012

518	200	197	4	2012
519	200	210	2	2012
520	200	211	4	2012
521	200	92	2	2012
522	200	88	1	2012
523	200	168	10	2012
524	200	144	4	2012
525	200	167	3	2012
526	200	213	4	2012
527	200	204	58	2012
528	200	203	58	2012
529	200	116	756	2012
530	200	201	11	2012
531	52	124	264	2012
532	52	25	4	2012
533	52	59	131	2012
534	52	81	11	2012
535	52	216	18	2012
536	52	1	5	2012
537	52	51	17	2012
538	52	69	24	2012
539	52	219	39	2012
540	52	220	25	2012
541	52	127	77	2012
542	52	29	89	2012
543	52	168	12	2012
544	52	197	3	2012
545	52	83	17	2012
546	52	92	238	2012
547	116	139	150	2012
499	13	128	300	2012
498	13	117	150	2012
550	13	195	20	2012
549	13	166	200	2012
500	13	196	5	2012
551	13	151	100	2012
552	13	87	210	2012
553	116	13	100	2012
548	116	211	550	2012
554	116	116	200	2012
555	142	116	300	2012
556	142	211	700	2012

557	172	116	300	2012
558	172	211	700	2012
559	182	116	300	2012
560	182	211	700	2012
561	87	116	300	2012
562	87	211	700	2012
563	184	116	300	2012
564	184	211	700	2012
565	149	116	300	2012
566	149	211	700	2012
567	183	116	300	2012
568	183	211	700	2012
569	143	116	300	2012
570	143	211	700	2012
571	153	116	300	2012
572	153	211	700	2012
573	185	116	300	2012
574	185	211	700	2012
575	186	116	300	2012
576	186	211	700	2012
577	154	116	300	2012
578	154	211	700	2012
579	180	116	300	2012
580	180	211	700	2012
581	179	116	300	2012
582	179	211	700	2012
583	187	116	300	2012
584	187	211	700	2012
585	173	116	300	2012
586	173	211	700	2012
587	116	116	100	default
588	119	116	100	2012
589	151	116	300	2012
590	151	211	700	2012

\.

--

-- Data for Name: geo_bin; Type: TABLE DATA; Schema: public; Owner: [REMOVED]

--

COPY geo_bin (uid, outer_geo, inner_geo) FROM stdin;

58	34	59
59	34	61
60	34	62
61	46	63
62	46	64
63	46	65
64	33	73
66	73	75
70	73	79
71	79	80
72	76	81
73	73	82
74	82	83
75	82	84
76	76	85
77	76	86
78	76	87
67	73	76
79	47	88
80	40	89
81	48	90
82	22	91
83	46	92
84	34	93
85	48	94
86	47	95
87	47	96
34	30	34
88	34	97
89	47	98
90	40	99
91	20	100
92	31	101
93	25	102
94	31	103
95	24	104
96	33	105
97	33	106
98	106	107
99	46	\N
100	46	58
20	19	20

21	19	21
22	21	25
23	21	26
24	21	27
25	21	28
26	21	29
27	20	22
28	20	23
29	20	24
30	22	30
31	22	31
32	22	32
33	30	33
35	30	35
36	30	36
37	30	37
38	30	38
39	30	39
40	30	40
41	30	41
42	30	42
43	30	43
44	30	44
45	33	45
46	33	46
47	33	47
48	33	48
49	46	49
50	46	50
51	49	51
52	49	52
53	49	53
54	49	54
55	49	55
56	49	56
57	49	57

\\.

--

-- Data for Name: geo_loc; Type: TABLE DATA; Schema: public; Owner: [REMOVED]

--

COPY geo_loc (uid, name) FROM stdin;

73 Vancouver Island
75 North Island
79 Central Island
80 Nanaimo
81 Victoria
82 West Coast
83 Tofino
19 Earth
20 Western Hemi
21 Eastern Hemi
22 North America
23 Central America
24 South America
25 Europe
26 Asia
27 Africa
28 Australia
29 Pacific Rim
30 Canada
31 United States
32 Mexico
33 British Columbia
35 Saskatchewan
36 Manitoba
37 Yukon
38 Northwest Territories
39 Nunavut
40 Ontario
41 Quebec
42 Newfoundland and Labrador
43 Nova Scotia
44 Prince Edward Island
45 Northern B.C.
46 Central B.C.
47 Southern B.C.
48 Coastal B.C.
49 Robson Valley
50 Prince George
51 Dome Creek
52 Crescent Spur

53 [REMOVED]
54 Dunster
55 Tete Jaune Cache
56 Valemount
57 Clearwater
58 Vanderhoof
59 Hinton
61 Jasper
62 Edmonton
63 Williams Lake
64 Quesnel
65 Salmon Arm
84 Ucluelet
85 Malahat
86 Mill Bay
87 Duncan
76 South Island
88 Kamloops
89 Ottawa
90 Vancouver
91 Generic_NorthAm
92 Vernon
93 Stony Plain
94 North Vancouver
95 Armstrong
96 Grindrod
34 Alberta
97 Calgary
98 Kaslo
99 Toronto
100 Generic_WstrnHemi
101 Generic_USA
102 Generic_Europe
103 California
104 Brazil
105 Generic_BC
106 Western B.C.
107 Houston
\.

--

```
-- Data for Name: nodes; Type: TABLE DATA; Schema: public; Owner: [REMOVED]
--

COPY nodes (uid, name, address1, address2, geo_loc, industry) FROM stdin;
[REMOVED]
\.

--
-- Name: edges_uid_key; Type: CONSTRAINT; Schema: public; Owner: [REMOVED]; Ta-
blespace:
--

ALTER TABLE ONLY edges
    ADD CONSTRAINT edges_uid_key UNIQUE (uid);

--
-- Name: geo_bin_uid_key; Type: CONSTRAINT; Schema: public; Owner: [REMOVED]; Ta-
blespace:
--

ALTER TABLE ONLY geo_bin
    ADD CONSTRAINT geo_bin_uid_key UNIQUE (uid);

--
-- Name: geo_loc_uid_key; Type: CONSTRAINT; Schema: public; Owner: [REMOVED]; Ta-
blespace:
--

ALTER TABLE ONLY geo_loc
    ADD CONSTRAINT geo_loc_uid_key UNIQUE (uid);

--
-- Name: node_uid_key; Type: CONSTRAINT; Schema: public; Owner: [REMOVED]; Ta-
blespace:
--

ALTER TABLE ONLY nodes
    ADD CONSTRAINT node_uid_key UNIQUE (uid);
```



```
--  
-- Name: unique_name; Type: CONSTRAINT; Schema: public; Owner: [REMOVED]; Ta-  
blespace:  
--  
  
ALTER TABLE ONLY geo_loc  
    ADD CONSTRAINT unique_name UNIQUE (name);  
  
--  
-- Name: public; Type: ACL; Schema: -; Owner: postgres  
--  
  
REVOKE ALL ON SCHEMA public FROM PUBLIC;  
REVOKE ALL ON SCHEMA public FROM postgres;  
GRANT ALL ON SCHEMA public TO postgres;  
GRANT ALL ON SCHEMA public TO PUBLIC;  
  
--  
-- PostgreSQL database dump complete  
--
```